

SOLUTION (An Integral)

Let $u = x^{1/3}$. Then $du = (1/3)x^{-2/3}dx$, so $dx = 3x^{2/3}du = 3u^2du$.

The integral then becomes $\int 3u^2e^u du$. This can be integrated by column integration or by using integration by parts twice, with the result $3u^2e^u - 6ue^u + 6e^u + C = 3(x^{2/3} - 2x^{1/3} + 2)e^{\sqrt[3]{x}} + C$.

SOLUTION (Speckled Eggs)

If he had one more hen, he would have a total of $65 + 1 = 66$ hens, of which one-third, or 22, are speckled, and one-half of these, or 11, would lay speckled eggs. Every $3/2$ hens lays an egg and a half in a day and a half, or one egg per day. Therefore, every hen lays $2/3$ of an egg per day (more realistically, 2 eggs every 3 days), and 11 hens lay $11 \times 2/3 = 22/3$ eggs per day (or 44 eggs, almost 4 dozen, after 6 days). To have 4 dozen eggs requires $4 \times 12 / (22/3) = 72/11 \approx 6.54$ days, so after 7 full days he will have 4 dozen speckled eggs.