## Three Roots

The given equation is

$$Ax^3 + (2 - A)x^2 - x - 1 = 0.$$

Notice that $x = 1$ is a solution for every possible $A$. Factoring yields $(x - 1)(Ax^2 + 2x + 1) = 0$. Using the quadratic formula, we find that the other two solutions are

$$x = \frac{-2 \pm \sqrt{4 - 4A}}{2A} = \frac{-1 \pm \sqrt{1 - A}}{A}.$$

Note that for $A > 1$, the discriminant is negative, and the roots are not real. If $A = 1$, this formula gives $-1$ as a double root.

We can also have a double root if

$$\begin{aligned} \frac{-1 \pm \sqrt{1 - A}}{A} &= 1 \\ \pm\sqrt{1 - A} &= A + 1 \\ 1 - A &= A^2 + 2A + 1 \\ 0 &= A^2 + 3A \\ 0 &= A(A + 3) \end{aligned}$$

So either $A = 0$ (impossible) or $A = -3$, in which case the other root is -$\frac{1}{3}$. So the possible values of the triple are $(1, -1, 1)$ and $(-3, 1, -\frac{1}{3})$.

## Correct Solutions

- Jim Manning
- Travis Williams (One correct triple.)
- Wes Geddings
- Joshua Hendrickson (One correct triple.)
- Holly Watson (One correct triple.)
- Kelvin Pompey (One correct triple.)
- James Austin Smith
- Oliver Gothe
- Travis Goldie (One correct triple.)
- Kayla Boatwright (One correct triple.)
- Kevin Ludwick
- Amber Moore (One correct triple.)
- Andrew Shore