NOVEMBER/DECEMBER SOLUTIONS

REMAINDER OF A POLYNOMIAL

Let $p(x) = x^{2011} + x^{1783} - 3x^{1707} + 2x^{341} + 3x^2 - 3.$ Without using a calculator or computer, find the remainder when $p(x)$ is divided by $x^3 - x$. Show all work on this problem.

SOLUTION

By the division algorithm we can write

$$p(x) = \left(x^3 - x\right)q(x) + r(x)$$

where $r(x)$ is the remainder term. Since $x^3 - x$ has degree 3, the remainder $r(x)$ must have a degree of at most two. Thus there exists integers a, b, and c such that $r(x) = ax^2 + bx + c$. Now, in order to determine a, b, and c, we let x be -1, 0, and 1 to obtain the following system of equations

$$-1 = p(-1) = 0 \cdot q(-1) + r(-1) = a - b + c$$
$$-3 = p(0) = 0 \cdot q(-1) + r(0) = c$$
$$1 = p(1) = 0 \cdot q(1) + r(1) = a + b + c$$

By solving the linear system we find that $a = 3$, $b = 1$, and $c = -3$. Therefore the remainder is $r(x) = 3x^2 + x - 3$

CORRECT SOLUTIONS

(1) Cole Franks
(2) Daniel Grier
(3) Reid Harris