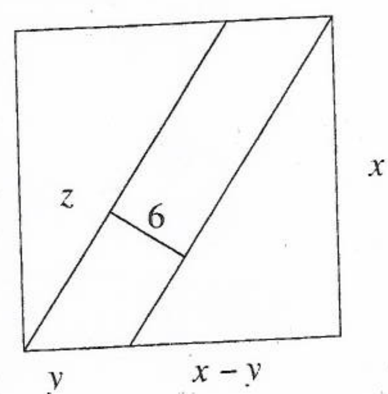
SOLUTION (A Trisected Square)

Let the length of the side of the square be x and the length of the short segment on the side be y, as shown, so the long segment on the side has length $x - y$, and the diagonal line has length $z = \sqrt{x^2 + (x-y)^2}$. Also, since the area of the parallelogram is 1/3 the area of the square, $xy = x^2/3$, so $y = x/3$. Thus $z = \sqrt{x^2 + (2x/3)^2} = \sqrt{13}x/3$. Also, the area of the parallelogram is $6z = 6\sqrt{13}x/3 = x^2/3$, so $x = 6\sqrt{13}$, and the area of the square is 468 square inches.



SOLUTION (An Infinite Sum)

Let $S(x) = \sum_{k=1}^{\infty} x^k$, which converges to $x/(1-x)$ if $|x| < 1$.

$$S'(x) = \sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}.$$

Multiply by x to get

$$\sum_{k=1}^{\infty} kx^k = \frac{x}{(1-x)^2}.$$

$$S''(x) = \sum_{k=1}^{\infty} k(k-1)x^{k-2} = \sum_{k=1}^{\infty} k^2 x^{k-2} - \sum_{k=1}^{\infty} kx^{k-2} = \frac{2}{(1-x)^3}.$$

Multiplying by x^2,

$$\sum_{k=1}^{\infty} k^2 x^k = \sum_{k=1}^{\infty} kx^k + \frac{2x^2}{(1-x)^3} = \frac{x}{(1-x)^2} + \frac{2x^2}{(1-x)^3}.$$

Letting $x = 1/7$ yields a sum of $7/27$.