SOLUTION (Reversed Digits as Ages)

In general, if ages are exactly reversed then the ages are AB and BA. A trivial solution is when A=B; that is, both people are the same age, namely one of the following ages: 00, 11, . . . , 99.

For other solutions, we'll assume A > B. Then the difference of the two ages is $(10A + B) - (10B + A) = 9(A\text{-}B)$. Consequently, the difference between the ages must be a multiple of 9, with $B \geq 0$.

The exact reversal occurs every 11 years: in 11 years, ages AB and BA become (A+1)(B+1) and (B+1)(A+1). Note that if A=9, then an additional 11 years moves us beyond age 100 and out of the considered range.

More specifically, let k = A-B. Then the age reversals occur as follows:
0A + 11x, for x = 0, 1, . . , (9-k)
A0 + 11x, for x = 0, 1, . . , (9-k)

A=1, B=0	A=2, B=0	A=3, B=0	A=4, B=0	A=5, B=0	A=6, B=0	A=7, B=0	A=8, B=8	A=9, B=0
01, 10	02, 20	03, 30	04, 40	05, 50	06, 60	07, 70	08, 80	09, 90
12, 21	13, 31	14, 41	15, 51	16, 61	17, 71	18, 81	19, 91	
23, 32	24, 42	25, 52	26, 62	27, 72	28, 82	29, 92		
34, 43	35, 53	36, 63	37, 73	38, 83	39, 93			
45, 54	46, 64	47, 74	48, 84	49, 94				
56, 65	57, 75	58, 85	59, 95					
67, 76	68, 86	69, 96						
78, 87	79, 97							
89, 98								

Note: Column 3 gives the solution to Part 1.

Correct Solutions (to at least one part):

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