SOLUTION (An Integral)

$$\int \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\,dx = \int \frac{(\sqrt{1+x}+\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}\,dx$$

$$= \int \frac{2+2\sqrt{1-x^2}}{2x}\,dx$$

$$= \int \frac{1}{x}\,dx + \int \frac{\sqrt{1-x^2}}{x}\,dx$$

In the second integral, let $x = \sin\theta$, $dx = \cos\theta\,d\theta$, and $\sqrt{1-x^2} = \cos\theta$.

$$\int \frac{1}{x}\,dx + \int \frac{\sqrt{1-x^2}}{x}\,dx = \ln|x| + \int \frac{\cos^2\theta}{\sin\theta}\,d\theta$$

$$= \ln|x| + \int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$$

$$= \ln|x| + \int \csc\theta - \sin\theta\,d\theta$$

$$= \ln|x| + \ln|\csc\theta - \cot\theta| + \cos\theta + C$$

$$= \ln|x| + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + \sqrt{1-x^2} + C$$

$$= \ln\left|1-\sqrt{1-x^2}\right| + \sqrt{1-x^2} + C$$

Note: There are many equivalent forms of this final answer. Some of them, for example, use inverse hyperbolic functions.

SOLUTION (Latin Squares)

There are $4! = 24$ ways to fill in the first row with 1 through 4. In the column that has a 1, there are then $3! = 6$ ways to fill in 2 through 4.

Next, consider the cell in the column and row containing 2. If either a 3 or 4 is put into that cell, then there is only one way to fill in the rest of the matrix. If a 1 is put into that cell, there are two ways to fill in the rest of the matrix. Altogether, then, there are 4 ways to finish the matrix.

Thus, the total number of distinct 4×4 Latin squares is $24 \times 6 \times 4 = 576$.