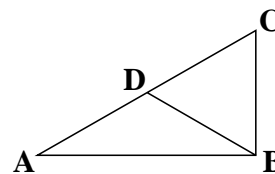


High School Math Contest
University of South Carolina
January 19, 2002

1. Which of the following numbers is equal to the sum

$$8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 + 8^8 ?$$

- (a) 8^8 (b) 8^9 (c) 64^8 (d) 8^{64} (e) 64^{64}
2. A line through the points $(m, -9)$ and $(7, m)$ has slope m . What is the value of m ?
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
3. In the figure shown, each edge of $\triangle BCD$ has length 1, D lies on \overline{AC} , and $\angle ABC = 90^\circ$. Find AB .



- (a) 1 (b) $3/2$ (c) $\sqrt{2}$ (d) $\sqrt{3}$ (e) 2
4. On a certain test, the average score for the women in the class is 83, while the average score for the men in the class is 71. If the average score of all the students in the class is 80, then what percentage of the students are women?
- (a) 60% (b) 65% (c) 70% (d) 75% (e) 80%
5. Let $3^a = 4$, $4^b = 5$, $5^c = 6$, $6^d = 7$, $7^e = 8$, and $8^f = 9$. What is value of the product $abcdef$?
- (a) 1 (b) 2 (c) $\sqrt{6}$ (d) 3 (e) $10/3$
6. How many different ways can we arrange the letters of SIX in a row? (For example, two such ways are IXS and SIX.)
- (a) 6 (b) 8 (c) 10 (d) 12 (e) 16

7. This written test has 30 multiple-choice questions. You will receive 5 points for each correct answer, 1 point for each answer left blank, and 0 points for each incorrect answer. Suppose that at the end of today's written test, five students make the following statements:

Alex says, "My test score is 147."

Blair says, "My test score is 144."

Chris says, "My test score is 143."

Drew says, "My test score is 141."

Erin says, "My test score is 139."

Only one of the students could possibly be correct. Which one?

- (a) Alex (b) Blair (c) Chris (d) Drew (e) Erin
8. The perimeter of a right triangle is 40 and the sum of the squares of its sides is 578. Find the length of the smallest side.
- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10
9. Let $f(x)$ be a function which contains 2 in its domain and range. Suppose that $f(f(x)) \cdot (1 + f(x)) = -f(x)$ for all numbers x in the domain of $f(x)$. What is the value of $f(2)$?
- (a) -1 (b) $-3/4$ (c) $-2/3$ (d) $-1/4$ (e) 0
10. Three different nonzero numbers, a , b , and c , are chosen so that
- $$\frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b}.$$
- What is the common value of these three quotients?
- (a) 3 (b) 1 (c) 0 (d) -1 (e) -3
11. If x and y are distinct numbers such that $2002 + x = y^2$ and $2002 + y = x^2$, what is the value of xy ?
- (a) -2001 (b) -1001 (c) -1 (d) 1 (e) 2000

12. Suppose that α is a root of $x^4 + x^2 - 1$. What is the value of $\alpha^6 + 2\alpha^4$?
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

13. An operation on a row of seven circles, where each circle is either black or white, consists of choosing any two of the circles and changing the colors of each of them (i.e., from black to white, or from white to black).



Which of the following rows of circles cannot be obtained by any repeated application of such operations upon the row of circles shown above?

- (a) ○ ● ○ ○ ● ○ ●
 (b) ● ○ ● ● ● ● ○
 (c) ● ● ● ● ● ● ●
 (d) ○ ○ ○ ○ ● ● ●
 (e) ● ○ ● ○ ● ○ ●

14. How many of the solutions to the following equation are negative?

$$x^4 - 6x^2 + 9 = 3x^3 + 5x$$

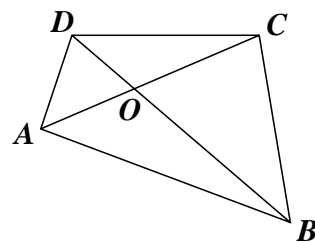
- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

15. How many rational roots does the following polynomial have?

$$2x^4 + 7x^3 + 9x^2 + 7x + 2$$

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

16. Let $ABCD$ be the quadrilateral shown, where O is the point of intersection of the line segments \overline{AC} and \overline{BD} . If the area of $\triangle AOB$ is 3, the area of $\triangle BOC$ is 6, and the area of $\triangle COD$ is 2, then what is the area of $\triangle DOA$?



- (a) 0.5 (b) 1 (c) 1.5 (d) 2 (e) 2.5

17. What is the sum of all solutions to the following equation?

$$x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}}}}}}$$

- (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

18. Suppose that a , b , c , and d are numbers which satisfy $a + b + c + d = 10$, $(a + b)(c + d) = 16$, $(a + c)(b + d) = 21$, and $(a + d)(b + c) = 24$.

What is the value of $a^2 + b^2 + c^2 + d^2$?

- (a) 30 (b) 39 (c) 46 (d) 51 (e) 54

19. For an arbitrary real number x , we define $[x]$ to be the greatest integer less than or equal to x . Let a and b be positive real numbers such that $a \cdot [a] = 17$ and $b \cdot [b] = 11$. What is the value of $a - b$?

- (a) $1/3$ (b) $1/2$ (c) $9/17$ (d) $6/11$ (e) $7/12$

20. Find all real numbers a such that the inequality $ax^2 - 2x + a < 0$ holds for all real numbers x .

- (a) $a < -2$ (b) $a < -1$ (c) $a < 0$
 (d) $a < -2$ or $a > 2$ (e) $a < -1$ or $a > 1$

21. Suppose that the complex number α is a solution to the equation

$$x + \frac{1}{x} = 2 \cos \left(\frac{\pi}{1001} \right).$$

What is the value of $\alpha^{2002} + \frac{1}{\alpha^{2002}}$?

- (a) -2 (b) -1 (c) 0 (d) 1 (e) 2

22. What is the remainder of

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + 1000 \cdot 1000!$$

divided by 2002?

- (a) 0 (b) 1 (c) 1000 (d) 1001 (e) 2001

23. Tammy, John, and Martha were all born at noon on January 19th, but in different years. When Tammy was four years old, John was three times as old as Martha. When Martha was twice as old as Tammy, John was five times as old as Tammy. How old was Tammy when John was twice as old as Martha?

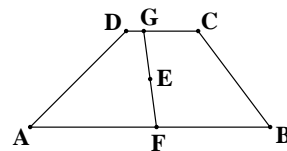
- (a) 8 (b) 12 (c) 16 (d) 20 (e) 24

24. Each square in the partially filled grid shown contains a number from 1 to 25, each number appearing exactly once. The numbers in each row and each column add up to 65. What is the value of $x + y$?

	7	25	18	
	21	19	y	
22	20			9
16			10	23
	x	6	24	

- (a) 15 (b) 17 (c) 19 (d) 21 (e) 23

25. The vertices of trapezoid $ABCD$ have coordinates $(-1, 3)$, $(9, 3)$, $(6, 7)$, and $(3, 7)$ respectively, and the point E has coordinates $(4, 5)$. A line through E intersects \overline{AB} at point F and \overline{CD} at point G . Find the area of trapezoid $AFGD$.



- (a) 12 (b) 13 (c) 14 (d) 15 (e) 16

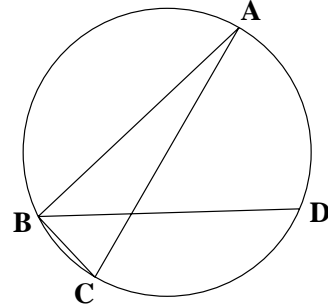
26. There is one real root α of $4x^{100} - 2x^2 + 3x - 1$ satisfying $0.15 < \alpha \leq 0.65$. Which of the following is nearest in value to α ?

- (a) 0.2 (b) 0.3 (c) 0.4 (d) 0.5 (e) 0.6

27. Find the number of integers n such that $\frac{5n + 26}{2n + 3}$ is an integer.

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

28. Points A , B , C , and D lie on a circle with $AB = 4$, $BC = 2$, \overline{AC} is a diameter, and $\angle ABD = \angle CBD$. What is BD ?

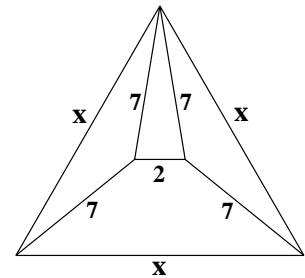


- (a) $2\sqrt{3} + 1$ (b) $\frac{9}{\sqrt{5}}$ (c) $3\sqrt{2}$ (d) $2 + \sqrt{5}$ (e) 4

29. How many integer pairs (a, b) are there with $1 \leq a \leq 100$ and $1 \leq b \leq 100$ such that the sum of $a + \sqrt{b}$ and its reciprocal is an integer?

- (a) 8 (b) 9 (c) 10 (d) 11 (e) 12

30. What is the value of x in the plane figure shown?



- (a) 12 (b) $7\sqrt{3}$ (c) 12.5 (d) $9\sqrt{2}$ (e) 13