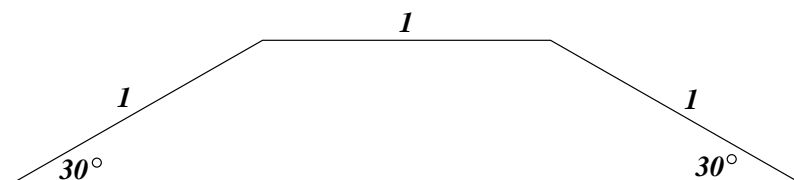


High School Math Contest
University of South Carolina
December 7, 1991

1. What is the area of the trapezoid below?



- (a) $\frac{2 + \sqrt{3}}{4}$ (b) 1 (c) 2 (d) $\frac{5 + 2\sqrt{3}}{2}$ (e) $\frac{7 - 2\sqrt{3}}{2}$
2. The value of $\frac{x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7}{x^{-3} + x^{-4} + x^{-5} + x^{-6} + x^{-7} + x^{-8} + x^{-9}}$ is
- (a) x^{20} (b) x^{16} (c) x^{10} (d) x^{-7} (e) x^{-2}
3. As you may recall, the sum of the interior angles of a triangle is 180° . How many sides must a polygon have if the sum of the interior angles is 2520° ?
- (a) 12 (b) 14 (c) 16 (d) 28 (e) 42
4. An item for sale is marked down 20%. By what percent must it then be marked up in order to return to the original selling price?
- (a) 20 (b) 22 (c) 23 (d) 24 (e) 25
5. Suppose that the number of elements in the set S is 105 and that S is split into n subsets of $11m + 2$ elements each. If m is an integer, then m is
- (a) 2 (b) 3 (c) 4 (d) 5 (e) 6
6. Suppose five ordinary dice are rolled. What is the probability that at least one 6 appears?
- (a) $1 - \left(\frac{5}{6}\right)^5$ (b) $\left(\frac{1}{6}\right)^5$ (c) $\left(\frac{5}{6}\right)^5$ (d) $\frac{1}{6}$ (e) $\frac{5}{6}$

7. Suppose $\cos(\theta) + \cos(2\theta) = 0$ and $0 \leq \theta \leq \pi/2$. Then $\cos(\theta) =$
- (a) 0 (b) 1 (c) $1/2$ (d) $\sqrt{3}/2$ (e) $\sqrt{2}/2$
8. Suppose that 1 and 2 are roots of $x^3 + ax^2 + bx + c = 0$ and that $a + b = -15$. Then $a =$
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
9. In a plane, points A and B are on one side of line ℓ . If A and B are both 3 centimeters from ℓ and are 4 centimeters from each other, find the radius of the circle that is tangent to ℓ and goes through A and B .
- (a) $13/6$ cm (b) $7/3$ cm (c) $15/6$ cm (d) $8/3$ cm (e) $17/6$ cm
10. $\sqrt{3 + \sqrt{2}} + \sqrt{3 - \sqrt{2}} =$
- (a) $\sqrt{6}$ (b) $2\sqrt{3}$ (c) $\sqrt{3 + \sqrt{3 - \sqrt{2}}}$ (d) 3.5 (e) $\sqrt{6 + \sqrt{28}}$
11. The largest solution of $x^6 - x^5 - x^4 - x^3 - x^2 - x - 1 = 0$ is in which of the following intervals?
- (a) $[0, 1)$ (b) $[1, 2)$ (c) $[2, 3)$ (d) $[3, 4)$ (e) $[4, \infty)$
12. For how many positive integer values of n is $1! + 2! + 3! + \cdots + n!$ a perfect square?
- (a) none (b) 1 (c) 2 (d) 3 (e) infinitely many

13. Suppose that the array of squares below is filled with positive integers in such a way that the product of each row, column, and each of the two main diagonals is the same. What must be the value in the lower right-hand square?

	36	2
	6	
		???

- (a) 8 (b) 12 (c) 16 (d) 24 (e) 36
14. Both solution X and solution Y contain alcohol and water. In solution X , the ratio of alcohol to water is 3:2. When equal amounts of solution X and solution Y are mixed, the ratio of alcohol to water is 3:4. What is the ratio of alcohol to water in solution Y ?
- (a) 1:1 (b) 9:26 (c) 10:25 (d) 10:24 (e) none of these
15. It is O 's move in the Tic-Tac-Toe game below. Which of the moves 1, 2, 3, 4, or 5 below is the best move for O ?

X	O	1
2	3	4
5	X	

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
16. The largest integer k such that 5^k divides $3(10!) + 12(5!) + 4(7!)$ is
- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

17. Suppose that t is an integer, $1 \leq t \leq 100$, and that the equation $x^2 + tx + \left(\frac{t^2 - t}{4}\right) = 0$ has an integral root. Then the number of possible values of t is:

- (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

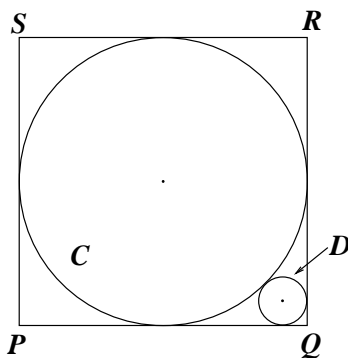
18. $\tan(2 \arcsin(x)) =$

- (a) $\frac{2\sqrt{1-x^2}}{1-2x^2}$ (b) $\frac{2x\sqrt{1-x^2}}{1-2x^2}$ (c) $\frac{2x\sqrt{1+x^2}}{1+2x^2}$
 (d) $\frac{x\sqrt{1-x^2}}{1-2x^2}$ (e) $\frac{2\sqrt{1+x^2}}{1+2x^2}$

19. $(\sin 20^\circ)(\tan 10^\circ + \cot 10^\circ)$ is equal to

- (a) $3/2$ (b) 3 (c) $1/3$ (d) 2 (e) $1/2$

20. Circle C is inscribed in square $PQRS$, and the length of segment PQ is 1. Inside square $PQRS$, circle D is tangent to circle C , segment PQ and segment QR . Find the radius of circle D .



- (a) $\frac{3-2\sqrt{2}}{2}$ (b) $\frac{\sqrt{2}-1}{1+2\sqrt{2}}$ (c) $\frac{\sqrt{2}-1}{2}$ (d) $\frac{\sqrt{2}+1}{8}$ (e) $2-\sqrt{2}$

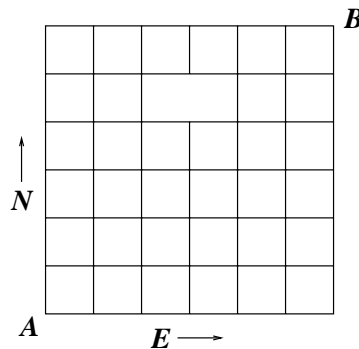
21. Which of the following is equal to $\sum_{k=1}^{99} \log_{10} \left(\frac{k+1}{k}\right)$?

- (a) 0 (b) 1 (c) 2 (d) $\log_{10} 99$ (e) $\log_{10}(100/99)$

22. What is the base of the numeral system in which $1/5 = .333\dots$?

- (a) 7 (b) 9 (c) 11 (d) 14 (e) 16

23. If S is the set of all points z in the complex plane such that $(3 + 4i)z$ is a real number, then S is a
- (a) line (b) triangle (c) circle (d) hyperbola (e) parabola
24. How many integers x are there such that $1 \leq x \leq 100$, and $x^3 + 4x + 2$ is divisible by 7?
- (a) 26 (b) 27 (c) 28 (d) 29 (e) 30
25. Moving along the line segments of this grid in the directions North and East only, how many different paths are there from A to B ? Notice that one segment is missing.



- (a) 320 (b) 342 (c) 356 (d) 382 (e) 390
26. If $16^{101} + 8^{101} + 4^{101} + 2^{101} + 1$ is divided by $2^{100} + 1$, then the remainder is
- (a) 0 (b) 2 (c) 4 (d) 11 (e) 101
27. Suppose that the straight line L meets the curve $y = 3x^3 - 15x^2 + 7x - 8$ in three points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) . Then $x_1 + x_2 + x_3 =$
- (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

28. Suppose that S is a subset of the integers that contains at least two elements and is such that:

For all integers x and y , if $x \in S$ and $y \notin S$, then $x + y \notin S$.

How many of the following statements are true about S ?

- (i) $0 \in S$.
 - (ii) If $x \in S$, then $-x \in S$.
 - (iii) If $x \in S$ and $y \in S$, then $x + y \in S$.
 - (iv) S must be infinite.
- (a) none (b) exactly 1 (c) exactly 2 (d) exactly 3 (e) all

29. Find the maximum diameter of a circle that lies completely on or above the x axis, passes through the origin, and intersects the parabola $y = x^2$ only at the origin.

- (a) $1/2$ (b) $\sqrt{2}/2$ (c) $\sqrt{3}/2$ (d) 1 (e) $\sqrt{2}$

30. Suppose that x_1, x_2, \dots, x_{40} and y_1, y_2, \dots, y_{40} are two permutations of the integers $1, 2, \dots, 40$. Then the value of $\sum_{k=1}^{40} (2x_k + y_k)$ is

- (a) 2360 (b) 2420 (c) 2460 (d) 2520
(e) Cannot be determined from this information