

High School Math Contest
University of South Carolina
December 5, 1987

1. The only value of a for which the simultaneous equations

$$\begin{cases} 2x + 3y = 5 \\ x + ay = 2 \end{cases}$$

would have no solution is

- (a) 2 (b) 0 (c) $3/2$ (d) $1/2$ (e) $-1/2$
2. If $xy = 2$ and $x^2 + y^2 = 5$, then $\frac{x}{y} + \frac{y}{x} =$
- (a) 0 (b) $5/2$ (c) 1 (d) $\sqrt{5/2}$ (e) $2/5$
3. How many subsets of $\{a, b, c, d, e, f, g\}$ contain both a and b ?
- (a) 9 (b) 12 (c) 16 (d) 25 (e) 32
4. Which of statements I, II, III, and IV below are true?
- I. $\cos \theta = \cos (-\theta)$ for all angles θ
II. $\sin \theta = \sin (-\theta)$ for all angles θ
III. $\cos \theta = \cos (-\theta)$ for some angles θ
IV. $\sin \theta = \sin (-\theta)$ for some angles θ
- (a) I, III, and IV, but not II (b) II, III, and IV, but not I
(c) III and IV, but not I or II (d) I and III, but not II or IV
(e) I and IV, but not II or III
5. The largest solution of $2 \log_{10} x = \log_{10}(3x - 20) + 1$ is
- (a) 10 (b) $20/3$ (c) $23/3$ (d) 20 (e) 100

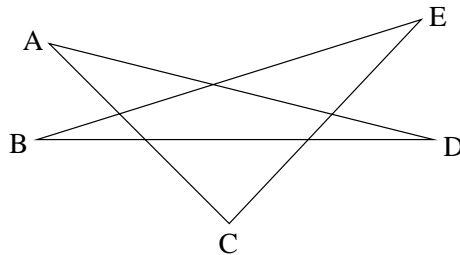
6. $\sqrt{9 + 4\sqrt{2}} =$

- (a) $3 + 2\sqrt{2}$ (b) $3 + 2\sqrt[4]{2}$ (c) $2 + 3\sqrt{2}$ (d) $2 + 3\sqrt[4]{2}$ (e) $1 + 2\sqrt{2}$

7. Trains leave from Philadelphia for Harrisburg every hour on the hour. The trip takes three hours. Each train waits at the Harrisburg depot one half hour and then returns to Philadelphia. The number of trains going the other way that it will pass on its return trip is

- (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

8. The sum of the measures of the angles A , B , C , D and E in the figure below is



- (a) 270° (b) 180° (c) 210° (d) 240° (e) 360°

9. Let $A = 1987 + \frac{1}{1987}$, $B = 1987 + \frac{1}{1987 + \frac{1}{1987}}$, and

$$C = 1987 + \frac{1}{1987 + \frac{1}{1987 + \frac{1}{1987}}}.$$

The numbers A , B , and C arranged in increasing order are

- (a) A, B, C (b) C, A, B (c) B, C, A (d) C, B, A (e) B, A, C

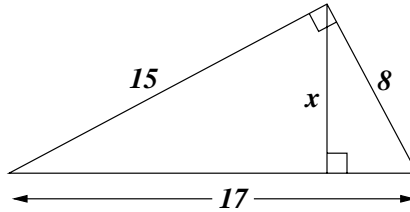
10. Assume that b and c are integers greater than one. In base b , c^2 is written as 10. Then b^2 , when written in base c , is

- (a) 100 (b) cannot be determined (c) 101 (d) 1010 (e) 10000

11. I have two dice, one red and one blue. When the two dice are rolled, the probability that the number showing on the red die is larger than the number showing on the blue die is

- (a) $1/2$ (b) $19/36$ (c) $2/3$ (d) $5/12$ (e) $4/9$

12. Find x in the figure below.



- (a) $120/17$ (b) 7 (c) $255/8$ (d) 8 (e) $\sqrt{2040}$

13. For a certain integer n , $5n + 16$ and $8n + 29$ have a common factor larger than one. That common factor is

- (a) 11 (b) 13 (c) 17 (d) 19 (e) 23

14. A certain function f satisfies $f(x) + 2f(6 - x) = x$ for all real numbers x . The value of $f(1)$ is

- (a) 3 (b) not possible to determine (c) 2 (d) 1 (e) -9

15. If the line $y = 2x + b$ is tangent to the circle $x^2 + y^2 = 1$, the values of b are

- (a) $\pm\sqrt{2}$ (b) $\pm\sqrt{2}/2$ (c) $\pm\sqrt{5}$ (d) $\pm\sqrt{5}/2$ (e) none of these

16. The 89-factor product

$$\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 87^\circ \tan 88^\circ \tan 89^\circ$$

is equal to

- (a) 1 (b) $\sqrt{3}$ (c) $1/2$ (d) $\pi/4$ (e) $3/4$

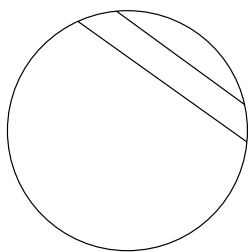
17. There are 120 five-digit numbers that can be formed by permuting 1, 2, 3, 4, and 5, such as

12345, 12354, 21435, . . . , 54321.

The sum of all these numbers is

- (a) 3,999,960 (b) 2,876,540 (c) 4,969,960
(d) 5,600,610 (e) 6,975,640

18. Two parallel chords in a circle have lengths 6 and 8. The distance between them is 1. Then the diameter of the circle is



- (a) 12 (b) 14 (c) 10 (d) 9 (e) $10\sqrt{3}$

19. $\sin(\cos^{-1}(\tan(\pi/6))) =$

- (a) $\sqrt{2/3}$ (b) $\sqrt{3}/2$ (c) $\sqrt{2}/2$ (d) $\pi/4$ (e) $1/2$

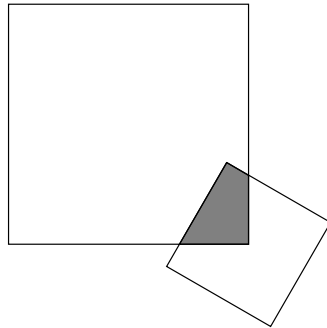
20. In base 10 arithmetic, the value of the digit d for which the number $d456d$ is divisible by 18 is

- (a) 0 (b) 2 (c) 4 (d) 6 (e) 8

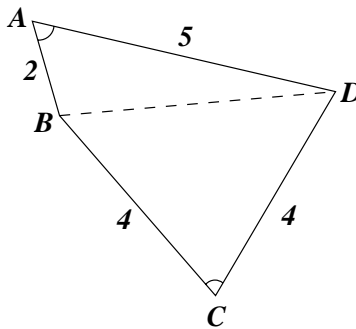
21. If $f(x)$ is a function that satisfies $f(2x + 1) = 2f(x) + 1$ for all x , and if $f(0) = 2$, then $f(3) =$

- (a) 5 (b) 9 (c) 11 (d) 13 (e) 15

22. A 5×5 square card is laid on a 10×10 square card so that the center of the small card lies directly over the corner of the large card. The area of the covered region is



- (a) $10\sqrt{2}/3$ (b) $75/14$ (c) $25/4$ (d) $3(1 + \sqrt{2})$ (e) 6
23. The largest number listed below which is less than $\sqrt{10020} - \sqrt{10010}$ is
- (a) 10 (b) 1 (c) $1/10$ (d) $1/20$ (e) $1/40$
24. Let L be a line and P be a point in the plane. Let c be a constant less than 1. The set of points Q such that the distance from P to Q is c times the distance from Q to L forms
- (a) a parabola (b) an ellipse (c) a hyperbola
 (d) half a hyperbola (e) a straight line
25. In quadrilateral $ABCD$, $AB = 2$, $BC = CD = 4$, $DA = 5$, and the opposite angles A and C are congruent. The length of the diagonal \overline{BD} is



- (a) $\frac{2}{3}\sqrt{30}$ (b) $2\sqrt{6}$ (c) 5 (d) $\sqrt{14}$ (e) $6\sqrt{2}$

26. The units digit (base 10) of $2^{(3^{456789})}$ is
- (a) 1 (b) 2 (c) 4 (d) 6 (e) 8
27. When $x^{100} - 2x^{99} + 4$ is divided by $x^2 - 3x + 2$, the remainder is
- (a) $x + 2$ (b) $x + 1$ (c) $2x + 1$ (d) $x - 1$ (e) $3x - 2$
28. A lattice point in the plane is a point whose x and y coordinates are integers. Let S be a set of lattice points in the plane such that for every pair of distinct points (x_1, y_1) and (x_2, y_2) in S , $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is not a lattice point. What is the biggest possible size of S ?
- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8
29. The sum of the squares of the roots of $x^3 + 2x^2 - 2x - 2$ is
- (a) 8 (b) 4 (c) 2 (d) 0 (e) -2
30. If x satisfies the equation $\tan x = \frac{\sin 10^\circ + \sin 40^\circ}{\cos 10^\circ + \cos 40^\circ}$, and x is between 0° and 90° , then x is equal to
- (a) 20° (b) 30° (c) 22.5° (d) 25° (e) 50°