MATH 550 Section 001 Fall 2005 Bonus Project 1

(Due in Class October 6)

Let a be the largest digit among the last four digits of your student number and b be the smallest positive digit. Consider the problem of finding the minimum value of $f(x, y) = \cos(x - y) + xy$ under the constraint

$$(*) \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(a) Plot the ellipse defined by $(*)$ together with the level curves of $f(x, y)$. You should use enough number of contours so that you can see roughly where the minimum should occur. (Helpful Maple commands: implicitplot, contourplot)

(b) Use the method of Lagrange multipliers to find the point(s) where f attains its minimum value under the constraint $(*)$. (Helpful Maple command: fsolve)

(c) Determine the minimum value of $f(x, y)$ under the constraint $(*)$.

Your solution should include a plot for (a) and answers to (b) and (c), with detailed explanations of how you obtained your answers. You should include a neat Maple worksheet containing the relevant computations.