Graphical Analysis in Polar Coordinates

Douglas Meade, Ronda Sanders, and Xian Wu

Department of Mathematics

Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. An animation in polar coordinates can be created easily with the `animatecurve` command.

Related Course Material/Preparation

- §11.3 and §11.4.
- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2+y^2} & \qquad x &= r\cos(\theta) \\ \tan\theta &= y/x & \qquad y &= r\sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- To identify and see some basic polar curves, you may want to check out the *PolarCurveID* and *Basic14Polar* maplets, which are available from the course website (last column in Lab M):

  `http://people.math.sc.edu/calclab/142L-S19/labs/`

- New Maple commands introduced in this lab include:

Command	Description
`arctan( y, x )`	Two-argument version of the inverse tangent. This is essentially equivalent to `arctan(y/x)` except that the signs of `x` and `y` are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument arctan ideal for converting from rectangular to polar coordinates.
`plot( ..., coords=polar);`	Plot a function in polar coordinates. Example: > `R :=t-> 2*cos(4*t);` > `plot(R(t), t=0..2*Pi, coords=polar);`
`animatecurve`	Animated sketch of a curve. Example: The limaçon $r = 1 + 3\sin(\theta)$ could be animated as follows: > `R :=t-> 1 + 3*sin( t );` > `animatecurve([R(t),t,t=0..2*Pi], coords=polar);` **Note:** *Need to execute* `with( plots ):` *before using* `animatecurve`.

Activities

1. Convert the following points to polar coordinates: (2,0), (3,3), (0,2), (-2,3), (-2,-5), (0,-3), (1,$-\sqrt{3}$). **Note:** *Compare the angles obtained with* `arctan(y/x)` *and* `arctan(y,x)`*.*
2. For each of the curves below:
   - Plot the curve in polar coordinates.
   - Animate the sketching of the curve.
   - When applicable, find the range that traces the curve exactly once.
     Note: *Optional arguments to the* `animatecurve` *command include:*
     - `frames=`*n creates an animation with n frames; the default is frames=16.*
     - `numpoints=`*n instructs Maple to use n points in each frame of an animation; the default is numpoints=50.*

(1)	$r = 2 + \sin(\theta)$	(2)	$r = \cos(3\theta)$	(3)	$r = \cos(4\theta)$
(4)	$r = 2 + \sin\left(\frac{5\theta}{3}\right)$	(5)	$r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$	(6)	$r = \sin\left(\frac{\theta}{5}\right)$
(7)	$r = 3(1 - \cos(\theta))$	(8)	$r = 1 + (\cos(\theta))^3$	(9)	$r = (\cos(\theta))^2$
(10)	$r = \ln(\theta)$	(11)	$r = \frac{\theta}{2}$	(12)	$r^2 = \sin(2\theta)$

3. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\dfrac{\theta}{4}\right)\right)^3$ is called the "butterfly curve". Plot and animate the curve to see why.

Working with Maple

```
> with(plots):
```

Activity 1:

```
> x:=-1;
> y:=3;
> r:=sqrt(x^2+y^2);
> theta:=arctan(y,x);
```

Activity 2:

```
> plot(2+sin(theta), theta=0...2*Pi, coords=polar);
> animatecurve([2+sin(theta),theta,theta=0..2*Pi],coords=polar);
```

Activity 3:

```
> r:=exp(cos(theta))-2*cos(4*theta)+(sin(theta/4))^3;
> plot(r,theta=0..8*Pi,coords=polar);
> animatecurve([r,theta,t=0..8*Pi],coords=polar,numpoints=350);
```

Assignment

There is no assignment this week.