Integration of Rational Functions by Partial Fractions

Douglas Meade, Ronda Sanders, and Xian Wu

Department of Mathematics

Overview

The objective of this lab is to use Maple to practice integration skills using partial fraction decomposition of rational functions. In this lab, we will explore writing a rational function (a quotient of polynomials) as a sum of simpler fractions, called partial fractions, which are easily integrated. For more complicated partial fractions, we may also need to complete the square before integrating.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
`convert(f(x), parfrac, x);` **Example:** `convert(x/((x^2+1)*(x-2)^3)),parfrac,x);`	Converts a rational function $f(x)$ into its partial fraction form.
`completesquare(f(x),x);` **Example:** `completesquare(x/(x^2+2*x+2),x);` **Note:** *You need to load the* `student` *package before using this command.*	Completes the square to rewrite a quadratic expression.

Note: *You may omit the last* `x` *in the above commands if the independent variable is obvious.*

- The *Partial Fractions: General Decomposition* maplet is available on the course website:

  http://people.math.sc.edu/calclab/142L-S19/labs → Partial Fractions: General Decomposition

  This maplet is designed to help you practice partial fraction decomposition. Click on **New Function** to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the **Include Completing the Square** box to make the decomposition more difficult.

- The *Partial Fractions: Evaluating the Integral* maplet is available on the course website:

  http://people.math.sc.edu/calclab/142L-S19/labs → Partial Fractions: Evaluating the Integral

  This maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on **Modify or Make Your Own Problem** to enter an integral. Click on **Show Partial Fraction Expansion**. Then evaluate the expanded integral and check your work.

Related course material

Thomas' Calculus §8.5: Integration of Rational Functions by Partial Fractions.

Activities

1. Launch the *Partial Fractions: General Decomposition* maplet from the course web page and do a few practice problems.

2. Follow the steps below to integrate the following rational functions:

(a) $f1(x) = \dfrac{1}{(x+a)(x+b)}$. (See steps below.)

(b) $f2(x) = \dfrac{2x+1}{x^2 + 2x + 1}$. (Page 487, ex. 12)

(c) $f3(x) = \dfrac{x^2}{(x+1)(x^2 + 2x + 1)}$. (Page 487, ex. 20)

(d) $f4(x) = \dfrac{2s+2}{(s^2 + 1)(s-1)^3}$. (Page 487, ex. 25)

(e) $f5(x) = \dfrac{x^2 - x + 4}{x^3 - 8}$.

(f) $f6(x) = \dfrac{9x^3 - 3x + 1}{x^3 - x^2}$. (Page 488, ex. 35)

(g) $f7(x) = \dfrac{y^4 + y^2 - 1}{y^3 + y}$. (Page 488, ex. 37)

(h) $f8(x) = \dfrac{x+4}{x^2 + 2x + 5}$.

Steps:

The following steps would be used to integrate $f1(x)$.

(a) Load the maple `student` package.
> `with(student):`

(b) Assign the function using the arrow notation.
> `f1 := x -> 1/((x+a)*(x+b));`

(c) Write down the general form for the partial fraction decomposition of the function by hand. Then use Maple to complete the expansion and compare it to your general form.
> `convert(f1(x), parfrac, x);`

(d) Use a pencil and paper to integrate the function. Then use Maple to complete the integration and check your answer.
> `int(f1(x),x);`

(e) You can also use the *Partial Fractions: Evaluating the Integral* maplet to check your integration.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the **beginning** of next week's lab.