

Integration By Parts and Trig Substitutions

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Overview

The objective of this lab is to provide some Maple tools to help you practice integration skills, such as integration by parts and trig substitutions.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
<code>Hint(Expression);</code> Examples: <code>Hint(Int(f(x),x));</code> Note: You need to load the <code>Student[Calculus1]</code> package before using this command.	returns <code>Hints</code> for a calculus problem matching the <code>Expression</code> which is typically in form of <code>Diff(...)</code> , <code>Int(...)</code> , or <code>Limit(...)</code> . Note: The first letter of <code>Hint</code> , <code>Diff</code> , <code>Int</code> , and <code>Limit</code> must be capitalized.
<code>Rule[rule](Expression);</code> Examples: <code>Rule[parts,U,V](Int(f(x),x));</code> <code>Rule[h1](Int(f(x),x));</code> <code>Rule[h1[2]](Int(f(x),x));</code>	used to apply a <code>rule</code> obtained from <code>Hint</code> to a matching problem. Note: It is often easier to use names and equation labels as inputs. In the last two examples, we assign our <code>hint</code> to <code>h1</code> .

Maple is very capable when evaluating integrals, derivatives, and limits, but if you want to see individual steps or methods to be applied for such calculus problems (rather than just final answers), you need to use commands `Hint` and `Rule`. However, there are a few things that you should know:

1. There are many different ways to do a calculus problem and Maple's way may be different from yours. While Maple in general will give good hints, you may find a different or better way to do the problem.
 2. Because of the constant C in integral formulas, your final answer to an indefinite integral may look different from the one given by Maple. The best way to be sure that the both are correct is to take the derivative of the difference to see if it equals zero.
 3. Maple may use methods that we have not learned yet.
 4. Maple should be used as a tool to facilitate your learning. Remember, you will need to be able to evaluate these integrals by hand. Do not rely too heavily on Maple.
- The *Integration by Parts* maplet is available from the course website:
<http://people.math.sc.edu/calclab/142L-S19/labs> → [Integration by Parts](#)

This maplet is designed to help you practice the steps involved in integration by parts. Its interface is very similar to the last maplet.

- The *Trig Substitution* maplet is available on the course website:
<http://people.math.sc.edu/calclab/142L-S19/labs> → [Trig Substitution](#)

This maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on **New Integral** to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click **Hint** or **Show** if you need help or want to skip a step.

Related course material

Thomas' Calculus: §8.2, §8.3 and §8.4.

Activities

1. Launch *Integration by Parts* maplet from the course web page and do a few practice problems.
2. Launch *Trig Substitution* maplet from the course web page and do a few practice problems. Remember you will need a pencil and paper to complete all the steps by yourself.
3. Use Maple to help you to evaluate the following:

(a) $\int x^5 e^{x^3} dx$ (Page 467, ex. 38)

(b) $\int e^\theta \sin \theta d\theta$ (Page 467, ex. 21)

(c) $\int x^3 \ln x dx$ (Page 467, ex. 6)

(d) $\int x \sin^{-1}(x^2) dx$

(e) $\int \frac{1}{\sqrt{9+x^2}} dx$ (Page 479, ex. 1)

(f) $\int \sin^3 x \cos^2 x dx$

(g) $\int \sin^2 x \cos^2 x dx$

Example Problem

Evaluate $\int x^5 e^{x^3} dt$.

1. Start a Maple session and don't forget to load `Student[Calculus1]` package.
`> with(Student[Calculus1]):`
2. Assign expression to be integrated as a function $f(x)$.
`> f1:=x -> x^5*exp(x^3);`
3. Apply `Hint` and assign the hint to `h1`.
`> h1:=Hint(Int(f1(x),x));`
 The return suggests to use parts with $U = x^3$ and $V = \frac{1}{3}e^{x^3}$ or a substitution with $U = x^3$. Notice that $U = x^5$ won't work for parts since $V = \int e^{x^3} dx$ cannot be evaluated.
4. To apply the first hint, enter the following:
`> Rule[h1](Int(f1(x),x));`
5. To apply the second hint, enter the following:
`> Rule[h1[2]](Int(f1(x),x));`
6. You should be able to complete the problem by hand after seeing the hint applied, but you may continue to receive and apply hints using the equation labels if you need to.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the **beginning** of next week's lab.