Series: Convergence Tests

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Overview

The intent of this lab is to introduce a maplet to provide additional practice determining the convergence or divergence of series.

Maple Essentials

- A link to the *SeriesConvergenceTestDrill* maplet can be found on the course website:

  `http://people.math.sc.edu/calclab/142L-S19/labs/` → SeriesConvergenceTestDrill

  The first hurdle in determining the convergence or divergence of a series is to select an applicable test. Once you have chosen a test, there are steps to be carried out, some of which could easily be overlooked. The best way (and the only way) to overcome these difficulties is to have a lot of practice and this maplet can be very helpful. In Step A, the maplet allows the user to either input series or have the maplet randomly generate one for practice. To obtain numerical evidence, the user can then choose a range of indices and plot terms and/or partial sums in Step B. In Step C, the user selects an applicable test to best of his/her knowledge. (You can move directly to Step C without plotting terms in Step B if you wish, and you can always choose a different test if your first choice in inconclusive.) Once the user selects a test, the maplet opens a new window and shows all the steps that need to be completed for that particular test. Be careful, the correctness of your result on the Comparison and Limit Comparison tests will depend on knowing whether the comparison series converges or diverges. If you need a reminder of the test that you are using, click `Hint`. This maplet is a great tool to check your work and answers for homework problems, but don't depend on it too much as you have to do problems on your own eventually.

Preparation

Sections 10.3 to 10.6 in Thomas' Calculus. In addition, review the basic qualitative properties of logarithms, powers, exponentials, and so on. For example, exponentials grow faster (at ∞) than polynomials, factorials grow faster than exponentials, and so on.

Assignment

With the help of the Maplet, work out the problems assigned by your lab instructor. Your assignment is due at the beginning of next weeks lab.

Activities

For each of the following series, decide first which test should be used in determining whether the series diverges or converges. Use *SeriesConvergenceTestDrill* maplet to carry out detailed steps. Try another test if your first choice is not applicable or the answer is inconclusive.

(1) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{2k+1}$	(2) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt{k^2-1}}{k^3+2k^2+5}$	(3) $\displaystyle\sum_{k=1}^{\infty}\frac{2^k k!}{(k+2)!}$
(4) $\displaystyle\sum_{k=2}^{\infty}\frac{1}{k\sqrt{\ln k}}$	(5) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k}{k+1}\right)^{k^2}$	(6) $\displaystyle\sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{k\ln k}$
(7) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{3k+1}$	(8) $\displaystyle\sum_{k=3}^{\infty}(-1)^k\frac{\ln k}{k}$	(9) $\displaystyle\sum_{k=1}^{\infty}\frac{k\cos(k\pi)}{k^2+1}$
(10) $\displaystyle\sum_{k=3}^{\infty}\frac{(2k)!}{4^k}$	(11) $\displaystyle\sum_{k=1}^{\infty}\frac{k^k}{k!}$	(12) $\displaystyle\sum_{k=1}^{\infty}\frac{k}{2^k}$
(13) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{3k+2}{2k-1}\right)^k$	(14) $\displaystyle\sum_{k=1}^{\infty}\frac{4k^2-2k+6}{8k^7+k-8}$	(15) $\displaystyle\sum_{k=1}^{\infty}\frac{\tan^{-1}(k)}{k^2}$