Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- The *DerivativeDrill*, *AntiDerivativeDrill*, and *IntBySub* Maplets are useful tools to help you practice your differentiation and integration skills. They are also useful to check any derivative or indefinite integral that you compute by hand. These maplets are available on the course website:

  `http://people.math.sc.edu/calclab/142L-S19/labs/`

- Maple commands introduced in this lab:

Command	Description
`:=x->`	define a function of x. eg: `f:=x->sqrt(x);` `g:=t->2*sin(t);`
`limit`	compute limits or sided limits: `limit(f(x),x=a);` finds the limit, $\lim\limits_{x \to a} f(x)$. `limit(f(x),x=a,right);` finds the one-sided limit from right, $\lim\limits_{x \to a^+} f(x)$. `limit(f(x),x=a,left);` finds the one-sided limit from left, $\lim\limits_{x \to a^-} f(x)$.
`diff`	compute derivatives of an expression: `diff(f(x), x);` finds the derivative of $f(x)$ with respect to x. `diff(f(x), x$n);` finds the nth derivative of $f(x)$.
`int`	compute definite and indefinite integrals: `int(f(x),x);` evaluates the indefinite integral, $\int f(x)\ dx$; `int(f(x),x=a..b);` evaluates the definite integral, $\int_a^b f(x)\ dx$.
`plot`	plot one or more functions on a specified window `plot(f(x), x=a..b);` plots the graph of $y = f(x)$ for $a < x < b$; `plot([f(x),g(x)], x=a..b);` graphs two functions in a single plot

  Note that Maple's **int** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions: Maple uses `sqrt(x)` for $\sqrt{x}$, `abs(x)` for $|x|$, `exp(x)` for e^x, and `x^n` for x^n. Other basic functions can be typed in as what they are but you must carefully use ( ) to group together and match up expressions as needed. You must also type in * when multiplication is presented. For example, you need to type in `sin(x)` instead of `sinx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$.

- Palettes: The **Expression** and **Calculus** palettes can be used instead of typing the full Maple commands for many functions and operations. The **Common Symbols** palette contains other symbols, including π (`Pi`) and ∞ (`infinity`).

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

Many of the problems below can be solved in Maple using different interface methods: typing in a command directly, using the context menu at the right, or using palettes. Try and/or combine different methods when possible. You should make it a habit to define a function or assign an expression to a name first. You should always inspect the output to verify that the function or expression was entered correctly. You can then apply operations by using the context menu at the right or referring to the name or the equation label (if you use commands or palettes).

1. Practice using the *DerivativeDrill*, *AntiDerivativeDrill*, and *IntBySub* Maplets on the course web-page.

2. Use Maple to compute the derivative of the following functions:

   (i) $f(x) = x^3 \ln(x)$ (ii) $g(x) = \dfrac{\arctan(x)}{\sqrt{x^2 + 1}}$ (iii) $h(x) = \dfrac{1 - \ln(x)}{1 + \ln(x)}$

   Notice that if you use the context menu to obtain the derivative the resulting derivative will be defined as a function. If you use the `diff` command (or the Calculus palette) the resulting derivative will be an expression.

3. Use Maple to compute the first, second, third, and 100th derivatives of $f(x) = x \sin(2x)$.

4. Use Maple to evaluate the following indefinite and definite integrals:

   (i) $\displaystyle \int \frac{x + 1}{(x - 2)^2} \, dx$ (ii) $\displaystyle \int_{-1}^{3} x e^{-x^2} \, dx$ (iii) $\displaystyle \int_{0}^{\pi/2} \left| \frac{1}{2} - \cos x \right| \, dx$

5. Use Maple to evaluate the following limits:

   (i) $\displaystyle \lim_{x \to 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2}$ (ii) $\displaystyle \lim_{n \to \infty} (1 + 3/n)^{2n}$ (iii) $\displaystyle \lim_{t \to 1^+} \tan\left(\frac{\pi}{2} t \right)$

6. Let

   $$f(x) = \frac{1}{2}x^4 - x^3 - 36x^2 + 108x$$

   In a Maple worksheet:

   (a) Define the function using proper Maple notation.

   (b) Use the context menu to find the first and second derivatives of $f(x)$. Assign the derivatives to names using `:=` with appropriate label references or by using the context menu.

   Hint: *Reasonable names for these quantities would be* `df` *and* `ddf`.

   (c) Plot the graph of $y = f'(x)$ on the interval $-10 < x < 10$ and estimate the intervals on which $y = f(x)$ is increasing.   **Note:** You should adjust the window to get a good picture of the graph.

   (d) Plot the graph of $y = f''(x)$ on the interval $-10 < x < 10$ and estimate the intervals on which $y = f(x)$ is concave up.

   (e) Create a single plot containing the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$ for $-10 < x < 10$. Did you estimate correctly in (d) and (e)?

   (f) Don't forget to give your graph a title and legend and make sure the curves can be identified on a black/white copy. You can achieve this using commands or by clicking on your graph to see the context menu. Remember if you use the context menu that your changes may be lost if you re-execute.

   (g) For some Maple assignments/projects it will be useful to copy your final plot to a text document such as the Word. See if you are able to do this now.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the **beginning** of next week's lab.