

Integration Methods II: Partial Fractions and Trig Substitutions

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Overview

The objective of this lab is to use Maple to help you practice your integration skills in partial fractions and trigonometric substitutions.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
<code>convert(f(x), parfrac, x);</code> Examples: <code>convert(x/((x^2+1)*(x-2)^3), parfrac);</code> <code>convert((a*t-b)/(t-c)^2, parfrac, t);</code>	Convert a rational function $f(x)$ into its partial fraction form. The variable name x (or whatever the name used) can be omitted if it is not ambiguous
<code>completesquare(f(x), x);</code> Examples: <code>completesquare(t/(t^2+2*t+2));</code> <code>completesquare(a*x^2+b*x+c, x);</code>	Complete squares. The variable name x (or whatever the name used) can be omitted if it is not ambiguous. Need to load the <code>student</code> package first.

- Three maplets *Trig Substitutions*, *Partial Fractions: General Decomposition*, and *Partial Fractions: Evaluating the Integral* are available from the course website (last column in Lab 7):

<http://www.math.sc.edu/calclab/142L-S08/labs>

1. The first maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on **New Integral** to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click **Hint** or **Show** if you need help or want to skip a step.
2. The second maplet is designed to help you practice partial fraction decomposition. Click on **New Function** to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the **Include Completing the Square** box to make the decomposition more difficult.
3. The third maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on **Modify or Make Your Own Problem** to enter an integral. Click on **Show Partial Fraction Expansion**. Then evaluate the expanded integral and check your work.

Related course material

§8.4 and §8.5 of the textbook.

Activities

1. Use Maple commands `convert` and `completesquare` to perform partial fractions for the following rational functions:

(a) $f1(x) = \frac{1}{x^2 - 6x - 7}$. (see ex. 10 on page 543)

(b) $f2(x) = \frac{x^5 - 4x^3 + 1}{x^3 - 4x}$. (see ex. 20 on page 544)

(c) $f3(x) = \frac{x^2}{(x+1)^3}$. (see ex. 25 on page 544)

(d) $f4(x) = \frac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)}$. (see ex. 30 on page 544)

(e) $f5(x) = \frac{x^4 + 6x^3 + 10x^2 + x}{x^2 + 6x + 10}$. (see ex. 32 on page 544)

(f) $f6(x) = \frac{x^2 + 1}{(x^2 + 2x + 3)^2}$. (see ex. 37 on page 544)

Instructions/Remarks:

- i. Do not forget to load the `student` package first:
`> with(student);`
 - ii. Define the function using the arrow notation, say,
`> f1:=x->1/(x^2-6*x-7);`
 - iii. Write down the general partial fraction decomposition form for each problem use a pencil and paper before applying the Maple command, so you can compare the results to make sure that you do understand key steps.
 - iv. You may want to define your own partial fraction operator, say `MyPF`, as follows:
`> MyPF:=f->completesquare(convert(f,parfrac,x),x);`
 You can then use it as a Maple command to do partial fractions (together with completing squares if needed) for a rational function in variable x (the variable name must be the same as the one used in the definition), say $f1(x)$ in activity 1, as follows:
`> MyPF(f1(x));`
2. Evaluate, by hand as much as you can, integrals of the above rational functions from their partial fractions. Use Maple to check or when you need help.
 3. Launch each of the *Trig Substitutions*, the *Partial Fractions: General Decomposition*, and the *Partial Fractions: Evaluating the Integral* maplets from the course web and do a few practice problems. Your TA will show you how to use them.

Assignment

Exercises 45 and 46 on page 536; exercises 38 and 39 on page 544.