Integration Methods II: Partial Fractions and Trig Substitutions

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Overview

The objective of this lab is to use Maple to help you practice your integration skills in partial fractions and trigonometric substitutions.

Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description

`convert(f(x), parfrac, x);`
Examples:
`convert(x/((x^2+1)*(x-2)^3),parfrac);`
`convert((a*t-b)/(t-c)^2,parfrac,t);`
	Convert a rational function $f(x)$ into its partial fraction form. The variable name x (or whatever the name used) can be omitted if it is not ambiguous

`completesquare(f(x),x);`
Examples:
`completesquare(t/(t^2+2*t+2));`
`completesquare(a*x^2+b*x+c,x);`
	Complete squares. The variable name x (or whatever the name used) can be omitted if it is not ambiguous. Need to load the `student` package first.

- Three maplets *Trig Substitutions*, *Partial Fractions: General Decomposition*, and *Partial Fractions: Evaluating the Integral* are available from the course website (last column in Lab 7):

  http://www.math.sc.edu/calclab/142L-S08/labs

  1. The first maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on **New Integral** to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click **Hint** or **Show** if you need help or want to skip a step.
  2. The second maplet is designed to help you practice partial fraction decomposition. Click on **New Function** to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the **Include Completing the Square** box to make the decomposition more difficult.
  3. The third maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on **Modify or Make Your Own Problem** to enter an integral. Click on **Show Partial Fraction Expansion**. Then evaluate the expanded integral and check your work.

Related course material

§8.4 and §8.5 of the textbook.

Activities

1. Use Maple commands `convert` and `completesquare` to perform partial fractions for the following rational functions:
   (a) $f1(x) = \dfrac{1}{x^2 - 6x - 7}$. (see ex. 10 on page 543)
   (b) $f2(x) = \dfrac{x^5 - 4x^3 + 1}{x^3 - 4x}$. (see ex. 20 on page 544)
   (c) $f3(x) = \dfrac{x^2}{(x+1)^3}$. (see ex. 25 on page 544)
   (d) $f4(x) = \dfrac{x^3 + x^2 + x + 2}{(x^2+1)(x^2+2)}$. (see ex. 30 on page 544)
   (e) $f5(x) = \dfrac{x^4 + 6x^3 + 10x^2 + x}{x^2 + 6x + 10}$. (see ex. 32 on page 544)
   (f) $f6(x) = \dfrac{x^2 + 1}{(x^2 + 2x + 3)^2}$. (see ex. 37 on page 544)

Instructions/Remarks:

   i. Do not forget to load the `student` package first:
   `> with(student):`
   ii. Define the function using the arrow notation, say,
   `> f1:=x->1/(x^2-6*x-7);`
   iii. Write down the general partial fraction decomposition form for each problem use a pencil and paper before applying the Maple command, so you can compare the results to make sure that you do understand key steps.
   iv. You may want to define your own partial fraction operator, say `MyPF`, as follows:
   `> MyPF:=f->completesquare(convert(f,parfrac,x),x);`
   You can then use it as a Maple command to do partial fractions (together with completing squares if needed) for a rational function in variable x (the variable name must be the same as the one used in the definition), say $f1(x)$ in activity 1, as follows:
   `> MyPF(f1(x));`

2. Evaluate, by hand as much as you can, integrals of the above rational functions from their partial fractions. Use Maple to check or when you need help.

3. Launch each of the *Trig Substitutions*, the *Partial Fractions: General Decomposition*, and the *Partial Fractions: Evaluating the Integral* maplets from the course web and do a few practice problems. Your TA will show you how to use them.

Assignment

Exercises 45 and 46 on page 536; exercises 38 and 39 on page 544.