Graphical Analysis in Polar Coordinates

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Overview

One of the most challenging aspects to polar coordinates is being able to visualize the graph of a polar function, $r = f(\theta)$. An animation showing exactly how the curve is traced out as the angle moves through its domain is even more useful than a static graph of the function.

The simplest polar plots can be created with the `plot` command — with one additional argument. To create an animation in polar coordinates it is easier to work with a *parametric form* of the equation. (Parametric curves will be discussed in more detail in Calculus III.)

Related Course Material/Preparation

- §11.1.
- Know the basic conversions between rectangular and polar coordinates:

$$\begin{aligned} r &= \sqrt{x^2+y^2} & \qquad x &= r\cos(\theta) \\ \tan\theta &= \tfrac{y}{x} & \qquad y &= r\sin(\theta) \end{aligned}$$

- Remember that all angles need to be specified in radians.
- Be prepared to create some surprising plots that would be almost impossible to create in rectangular coordinates.

Maple Essentials

- The *PolarCurveID* and *Basic14Polar* maplets are available from the course website (last column in Lab 14):

  `http://www.math.sc.edu/calclab/142L-S07/labs`

- New Maple commands introduced in this lab include:

Command	Description
`arctan( y, x )`	two-argument version of the inverse tangent this is essentially equivalent to `arctan(y/x)` except that the signs of `x` and `y` are used to extend the range from $(-\frac{\pi}{2}, \frac{\pi}{2})$ to $(-\pi, \pi)$; this modification makes the two-argument arctan ideal for converting from rectangular to polar coordinates
`plot( ...,` `coords=polar);`	plot a function in polar coordinates the most common usage is: `> R :=t-> 2*cos(4*t)` `> plot( R(t), t=0..2*Pi, coords=polar );`
`animatecurve`	animated sketch of a curve e.g., the limaçon $r = 1 + 3\sin(\theta)$ could be animated as follows: `> R :=t-> 1 + 3*sin( t );` `> animatecurve([R(t),t,t=0..2*Pi], coords=polar);` **Note:** *Execute* `with( plots ):` *before using* `animatecurve`.
`unassign`	remove assignments from a Maple name to prevent the name from evaluating to its value, it is necessary to enclose each name in single quotes, e.g., `> unassign( 'x', 'y', 'r' );`

Activities

1. Convert the following points to polar coordinates: (2,0), (3,3), (0,2), (-2,3), (-2,-5), (0,-3), (1,$-\sqrt{3}$). **Note:** *Compare the angles obtained with* `arctan(y/x)` *and* `arctan(y,x)`.
2. Create plots of the unit circle, $x^2 + y^2 = 1$, in both rectangular and polar coordinates. **Note:** *In which coordinate system is it easier to plot the unit circle?*
3. For each of the curves below:
   - Find a parameter interval that traces the curve <u>exactly</u> once.
   - Plot the curve in polar coordinates.
   - Animate the sketching of the curve.
     Hint: *A polar function $r = f(\theta)$ can be written in parametric form as $r = f(t)$, $\theta = t$.*
     Note: *Optional arguments to the* `animatecurve` *command include:*
     - `frames=`*num creates an animation with num frames; the default number of frames is 16.*
     - `numpoints=`*num instructs Maple to use num points in each frame of an animation; the default number of points is 50.*

(i)	$r = 2 + \sin(\theta)$	(ii)	$r = \cos(4\theta)$	(iii)	$r = 3(1 - \cos(\theta))$
(iv)	$r = \sin\left(\frac{\theta}{5}\right)$	(v)	$r = \sin(\theta) + \cos\left(\frac{\theta}{3}\right)$	(vi)	$r = 2 + \sin\left(\frac{5\theta}{3}\right)$
(vii)	$r = \ln(\theta)$	(viii)	$r = \frac{\theta}{2}$	(ix)	$r = 1 + (\cos(\theta))^3$
(x)	$r = (\cos(\theta))^2$	(xi)	$r^2 = \cos(2\theta)$		

4. The polar function $r = e^{\cos(\theta)} - 2\cos(4\theta) + \left(\sin\left(\frac{\theta}{4}\right)\right)^3$ is called the "butterfly curve".
   (a) Find a parameter interval that traces this curve <u>exactly</u> once.
   (b) Plot or animate the curve.

Assignment

- There is no assignment this week but you need to complete an end-of-course survey. Your TA has instructions for turning in the survey.
- You have just completed the last math-142 lab. Congratulations and have a great break!