

# Volume By Definite Integral

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## Overview

The objective of this lab is to use Maple to help you practice computing volumes using definite integrals. Several visual and interactive tools are introduced as follows.

## Maple Essentials

- The *Volumes of Revolution* tutor is started from the Maple 9.5 user interface under the Tools menu:

Tools → Tutors → Calculus - Single Variable → Volumes of Revolution...

This tutor plots, sets up, and calculates a definite integral for the volume of the revolution of a region bounded above by a curve  $y = f(x)$  and below by the  $x$ -axis from  $x = a$  to  $x = b$ , where  $f(x)$ ,  $a$ , and  $b$  are user-specified (be sure to enter them in the appropriate boxes). You may choose the **Horizontal Axis** (the  $x$ -axis) or the **Vertical Axis** (the  $y$ -axis) as the **Axis of Revolution** and the volume is calculated by the method of disks or the method of shells, respectively. It also shows the maple command used. This is a great tool to use when you need the plot of the solid or want to check your answer to a problem.

- The *Volumes of Revolution* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-S06/labs> → Volumes of Revolution (TAMU)

This maplet is designed to help you to practise or to carry out individual steps in computing the volume of the revolution of a computer generated or user-specified region (click on **New Problem** or on **Modify or Make Your Own Problem**, respectively). After getting or entering the problem, your first step (**Step 1a**) is to choose the appropriate method: **Disks**, **Washers**, or **Cylinders**. You then enter the integral for the volume using maple command in **Step 1b**. You may click on **Hint** or **Show** if you need help to get started. After setting up the correct integral, we may enter your evaluation to check it or let the maplet evaluate the integral for you by clicking on **Show** in **Step 2**. An important feature of this maplet is its visualization tool. Please click on different boxes in the **Visualization** window to try it out. You can even rotate the plot with your mouse.

- Another maplet *Volumes by Slicing* is available from the course website:

<http://www.math.sc.edu/calclab/142L-S06/labs> → Volumes by Slicing (TAMU)

This maplet is still under the development. It deals with volumes by the method of slicing and has the similar features as the other maplet just introduced. As of now, it can only handle solids with certain types of regions as its base. You may want to use it if it applies to a problem that you are interested.

- Important Maple command introduced in this lab:

Command/Example	Description
<code>VolumeOfRevolution(top, bot, x=a..b, options);</code> <b>Examples:</b> <code>VolumeOfRevolution(x,x^2,x=0..1,output=plot);</code> <code>VolumeOfRevolution(arcsin(x),0,x=0..1,output=integral);</code> <code>VolumeOfRevolution(x,x^2,x=0..1,axis=vertical,output=integral);</code> <b>Note:</b> You need to load the <code>Student[Calculus1]</code> package before using this command. So type in the following command first: <code>with(Student[Calculus1]):</code>	visualize a solid of revolution or find the definite integral corresponding to either the Methods of Washers or Shells.

The general syntax for *visualizing* a solid of revolution of a region bounded by  $y = top$  and  $y = bot$  from  $x = a$  to  $x = b$ :

```
VolumeOfRevolution(top, bot, x=a..b, output=plot);
```

To get *definite integral for the volume*, change `output=plot` to `output=integral` in the above command.

To evaluate the integral obtained, you can use `evalf` command:

```
evalf(%);
```

The default axis of revolution is the  $x$ -axis ( $y = 0$ ), and the volume is computed by the Method of Disks/Washers. If the axis of revolution is the  $y$ -axis ( $x = 0$ ), include `axis=vertical` as an addition argument in the above command. In this case, the volume is calculated by the Method of Shells. When the bottom edge of the region is  $y = 0$ , the second argument (*bot*) can be omitted, but this is not advised.

### Related course material/Preparation

§7.2 and §7.3 of the textbook.

### Activities

- Use the *Volume of Revolution* maplet to visualize solids of revolution formed by rotating regions bounded by the following curves about the  $x$ -axis. Also, use the maplet to test your formulation of the definite integral for the volume of the solid first by the method of disks/wishers and then by the method of cylindrical shells. Evaluate the integral if possible and use the maplet check your answer or ask maplet to do it if the integral is too hard to evaluate. See the introduction of the maplet on the first page for detailed steps. **Remember that the variable of integration and the axis of rotation are the same in the method of disks/washers but they are different in the method of shells. You also need to convert equations of curves to express  $x$  as functions of  $y$  if the variable of integration is  $y$ . Be sure to modify your entering of problems for different methods.** You may want to use `plot` first to graph the functions to get an idea of the region you are rotating.
  - $y = 0, y = 2 - x^2, x = 0, x = 1$
  - $y = \sin(x), y = \cos(x), x = 0, x = \frac{\pi}{4}$
  - $y = x^2, y = x^3$
  - $y = 1 - x^2, y = 2 + x^2, x = -1, x = 1$
- Repeat Activity 1 for solids of revolution formed by rotating regions bounded by the following curves about the  $y$ -axis.
  - $y = \sin(x), x = 0, y = 1$
  - $y = x^2, y = x^3$
  - $y = \ln(x), x = 1, y = 1$
- Repeat at least one example from each of Activity 1 and Activity 2 using the `VolumeOfRevolution` command in a Maple worksheet. Remember to execute the command: `with(Student[Calculus1]):` to load the package first.

### Remark:

Please skip the parts on the method of shells if your class only covered the method of disks/washers.

### Assignment

Complete lab activities and use the Maple to do exercises 45,46, and 47 on page 458.