

# Volume By Definite Integral

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## Overview

With different methods, formulations, and rotating axes, the volume of revolution (§7.2 and §7.3) can be a confusing topic at first. The objective of this lab is to introduce visual and interactive Maple tools to help your practice of such problems. Related Maple commands are also introduced.

## Note:

Please skip the parts on the method of shells if your class only covered the method of disks/washers.

## Maple Essentials

- The *Volumes of Revolution* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-F06/labs> → Lab 3 → last column.

This maplet is designed to help you to practise individual steps in computing the volume of revolution of a computer generated or user-specified region (click on **New Problem** or **Modify or Make Your Own Problem**, respectively). After getting or entering the problem, your first step (**Step 1a**) is to choose the appropriate method: **Disks**, **Washers**, or **Cylinders**. You then enter the integral for the volume in **Step 1b**. You may click on **Hint** or **Show** if you need help to get started. After setting up the correct integral, we may enter your evaluation to check it or let the maplet evaluate the integral for you by clicking on **Show** in **Step 2**. An important feature of this maplet is its visualization tool. Please click on different boxes in the **Visualization** window to try them out. You can even rotate the plot with your mouse.

This is also a great tool to use when you want to see the plot of the solid or want to check your answer to a problem. It shows not only the answer but also the formulation. However, you need to make sure that your region is entered correctly.

- Another maplet *Volumes by Slicing* is available from the course website:

<http://www.math.sc.edu/calclab/142L-F06/labs> → Lab 3 → last column.

This maplet is still under the development. It deals with volumes by the method of slicing and has the similar features as the other maplet just introduced. As of now, it can only handle solids with certain types of regions as its base. You may want to use it if it applies to a problem that you are interested.

- Important Maple command introduced in this lab:

Command/Example	Description
<code>VolumeOfRevolution(top, bot, x=a..b, options);</code> <b>Examples:</b> <code>VolumeOfRevolution(x,x^2,x=0..1,output=plot);</code> <code>VolumeOfRevolution(arcsin(x),0,x=0..1,output=integral);</code> <code>VolumeOfRevolution(x,x^2,x=0..1,axis=vertical,output=integral);</code> <b>Note:</b> You need to load the <code>Student[Calculus1]</code> package before using this command (see command with below).	visualize a solid of revolution of a region or find the definite integral for its volume
<b>with:</b> <b>Examples:</b> <code>with(Student[Calculus1]): with(plots):</code> <b>Note:</b> To execute a line without displaying the output, use <code>:</code> instead of <code>;</code> at the end.	load a special maple package.

The general syntax for *visualizing* a solid of revolution of a region bounded by  $y = top$  from the above and  $y = bot$  from the below over  $a \leq x \leq b$  is:

```
VolumeOfRevolution(top, bot, x=a..b, output=plot);
```

When the bottom edge of the region is the  $x$ -axis ( $y = 0$ ), the second argument ( $bot$ ) can be omitted, but this is not advised.

To get the *definite integral* for the volume instead of the plot, change the option `output=plot` to `output=integral` in the above command.

To evaluate the integral obtained, you can apply commands `value` or `evalf` or use the right clicking.

The default axis of revolution is the  $x$ -axis ( $y = 0$ ), and the volume is computed by the method of disks/washers. If the axis of revolution is the  $y$ -axis ( $x = 0$ ), include `axis=vertical` as an addition argument in the above command. In this case, the volume is calculated by the method of shells. If the axis of revolution is a line other than the  $x$ -axis or the  $y$ -axis, say,  $y = -1$ , include `distancefromaxis=-1` as an addition option in the above command.

### Activities

1. Use the *Volume of Revolution* maplet to test your formulation of the definite integral for the volume of revolution for each of solids formed from rotating regions bounded by the following curves about the  $x$ -axis. Evaluate the integral and check the answer if possible or ask the maplet to do the evaluation if it is too hard to do it by hand.
  - (a)  $y = 0$ ,  $y = 2 - x^2$ ,  $x = 0$ ,  $x = 1$ . The method of disks/washers should be used. Why?
  - (b)  $y = x^2$ ,  $y = 2 - x$ ,  $y = 0$ . The method of shells should be used. Why?
  - (c)  $y = x^2$ ,  $y = x^3$ . Try both methods.
  - (d)  $y = \sqrt{x}$ ,  $y = 2 - x$ ,  $y = 0$ . Which method should be used?
  - (e)  $y = \sqrt{x}$ ,  $y = 2 - x$ ,  $x = 0$ . Which method should be used?

#### General Steps:

- i. Sketch the region and find intersections if needed (use Maple if you have to). Identify the rotating axis.
  - ii. Decide which method to use and then choose the variable of integration accordingly. **The rule is to select the same variable of integration as the rotating axis with the method of disks/washers but the different variable with the method of shells.**
  - iii. Start the maplet and click on **Modify** or **Make Your Own Problem** to enter the problem. You need to covert equations of curves to express  $x$  as functions of  $y$  if the variable of integration is  $y$ . Be careful as the maplet only knows what you entered and won't verify such things as which curve is really on the top and so on.
  - iv. Select the choosing method (in **Step 1a**) and enter your integral to check (in **Step 1b**). If you need the general formula, click on **Hint**.
  - v. Evaluate the integral by hand and check or let the maplet do it for you. You may also want to try its visualization tool (see the introduction of the maplet on the first page).
2. Repeat Activity 1 for each of solids formed from rotating regions bounded by the following curves about the  $y$ -axis.
    - (a)  $y = \sin(x)$ ,  $y = \cos(x)$ ,  $x = 0$ ,  $x = \pi/4$ . The method of shells should be used. Why?
    - (b)  $y = \ln(x)$ ,  $x = 1$ ,  $y = 1$ . Try both methods. Which one is better?
    - (c)  $y = x^2$ ,  $y = x^3$ . Try both methods and compare them to (c) of Activity 1.
  3. Repeat at least one example from each of Activity 1 and Activity 2 using the `VolumeOfRevolution` command in a Maple worksheet. Remember to execute the command: `with(Student[Calculus1]):` to load the package first.

### Assignment

Complete lab activities and do the following exercises using Maple: 46 and 47 on page 458; 18 on page 464; 24 on page 465; 12 on page 508 (do evaluate).