Integration Methods II: Partial Fractions and Trig Substitutions

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Overview

The objective of this lab is to use Maple to help you practice your integration skills in partial fractions and trigonometric substitutions.

Maple Essentials

- Three maplets *Trig Substitutions*, *Partial Fractions: General Decomposition*, and *Partial Fractions: Evaluating the Integral* are available from the course website (last column in Lab 7):

    http://www.math.sc.edu/calclab/142L-F06/labs

    Those maplets are designed to help you to understand named methods following individual steps with randomly generated problems. In a near future, they will be able to take user-specified problems.

- Important Maple commands introduced in this lab:

Command/Example	Description

`convert(f, parfrac, `*`variablename`*`);`
Examples:
`convert(x/(x^2+1)*(x-2)^3),parfrac);`
`convert((a*t-b)/(t-c)^2,parfrac,t);`
	Convert a rational function f into its partial fraction form. The variable name can be omitted if it is not ambiguous

`completesquare(f,`*`variablename`*`);`
Examples:
`completesquare(x/(x^2+2*x+2));`
`completesquare(a*x^2+b*x+c,x);`
	Complete squares. The variable name can be omitted if it is not ambiguous. Need to load the `student` package first.

Related course material

§8.4 and §8.5 of the textbook.

Activities

1. Use Maple commands `covert` and `completesquare` to perform partial fractions for the following rational functions:

   (a) $f1 = \dfrac{1}{x^2 - 6x - 7}$. (see ex. 10 on page 543)

   (b) $f2 = \dfrac{x^5 - 4x^3 + 1}{x^3 - 4x}$. (see ex. 20 on page 544)

   (c) $f3 = \dfrac{x^2}{(x+1)^3}$. (see ex. 25 on page 544)

   (d) $f4 = \dfrac{x^3 + x^2 + x + 2}{(x^2 + 1)(x^2 + 2)}$. (see ex. 30 on page 544)

   (e) $f5 = \dfrac{x^4 + 6x^3 + 10x^2 + x}{x^2 + 6x + 10}$. (see ex. 32 on page 544)

   (f) $f6 = \dfrac{x^2 + 1}{(x^2 + 2x + 3)^2}$. (see ex. 37 on page 544)

Remarks:

   (a) Write down the partial fraction decomposition form for each problem by hand first, so you can compare the results with Maple to make sure that you do understand key steps.

   (b) Don't forget to include `with(student):` to load the `student` package.

   (c) You may want to define your own partial fraction operator, say `MyPF`, as follows:

   `> MyPF:=f->completesquare(convert(f,parfrac,x),x);`

   You can then use it as a Maple command to do partial fractions (with completing squares if needed) for a rational function in variable x, say $f1$ in activity 1, as follows:

   `> f1pf:=MyPF(f1);`

2. Evaluate, by hand as much as you can, integrals of the above rational functions from their partial fractions. Ask your TA or use Maple when you need help.

3. Launch each of the *Trig Substitutions*, the *Partial Fractions: General Decomposition*, and the *Partial Fractions: Evaluating the Integral* maplets from the course web and do a few practice problems. Your TA will show you how to use them.

Assignment

Exercises 45 and 46 on page 536; exercises 38 and 39 on page 544.