

Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- Maple commands introduced in this lab:

Command	Description
<code>:=</code>	assign a name to a quantity. eg: <code>f:=sqrt(x);</code> <code>eq1:=x+y=2;</code>
<code>limit</code>	compute limits or sided limits: <code>limit(F, x=a);</code> finds the limit, $\lim_{x \rightarrow a} F$; <code>limit(F, x=a, right);</code> finds the sided limit from right, $\lim_{x \rightarrow a^+} F$; <code>limit(F, x=a, left);</code> finds the sided limit from left, $\lim_{x \rightarrow a^-} F$.
<code>diff</code>	compute derivatives of an expression: <code>diff(f, x);</code> finds the derivative of f with respect to x , $\frac{df}{dx}$; <code>diff(f, x\$n);</code> finds the n th derivative of f , $\frac{d^n f}{dx^n}$
<code>int</code>	compute definite and indefinite integrals: <code>int(f, x);</code> evaluates the indefinite integral, $\int f dx$; <code>int(f, x=a..b);</code> evaluates the definite integral, $\int_a^b f dx$.
<code>plot</code>	plot one or more functions on a specified window <code>plot(f, x=a..b);</code> plots the graph of $y = f$ for $a < x < b$; <code>plot([f, g], x=a..b);</code> graphs two functions in a single plot

Note that Maple's `int` command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions: Maple uses `sqrt(x)` for \sqrt{x} , `abs(x)` for $|x|$, `exp(x)` for e^x , and `x^n` for x^n . Other basic functions can be typed in as what they are but you must carefully use `()` to group together and match up expressions as needed. You must also type in `*` when a multiplication is presented. For example, you need to type in `sin(x)` instead of `sindx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$. Finally, do not forget to type in `;` at the end of a line that is to be executed.
- Palettes: The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Symbol** palette contains other symbols, including π (Pi) and ∞ (infinity). To view both palettes simultaneously, drag one of the palettes to another edge of the Maple window.

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

Many (but not all) problems below can be solved in Maple using different interface methods: typing in a command directly, using the right-clicking, or using palettes. Try and/or combine different methods when possible. In any case, you should make it a habit to enter an expression and assign it to a name first. You should always inspect the output to verify that the expression was entered correctly. You can then apply operations by the right-clicking or referring the name or the equation label (if you use commands or palettes).

1. Use Maple to compute the derivative of the following functions:

$$(i) \quad f1 := x^3 \ln(x) \qquad (ii) \quad f2 := \frac{\arctan(x)}{\sqrt{x^2 + 1}} \qquad (iii) \quad f3 := \frac{1 - \ln(t)}{1 + \ln(t)}$$

2. Use Maple to compute the first and 100th derivatives of $x \sin(2x)$.

3. Use Maple to evaluate the following indefinite and definite integrals:

$$(i) \quad \int \frac{x + 1}{(x - 2)^2} dx \qquad (ii) \quad \int_{-1}^3 x e^{-x^2} dx \qquad (iii) \quad \int_0^{\pi/2} \left| \frac{1}{2} - \cos x \right| dx$$

4. Use Maple to evaluate the following limits:

$$(i) \quad \lim_{x \rightarrow 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2} \qquad (ii) \quad \lim_{n \rightarrow \infty} (1 + 3/n)^{2n} \qquad (iii) \quad \lim_{t \rightarrow 1^+} \tan\left(\frac{\pi}{2}t\right)$$

5. Let

$$f(x) = \ln\left(\frac{(x^2 + 2)^2}{\sqrt{4x^2 + 1}(x^2 + 1)}\right).$$

In a Maple worksheet:

- (a) enter the function and assign it to a name of your choice

Hint: Use a short, but meaningful, name, say f .

- (b) find and name the first and second derivatives of $f(x)$

Hint: Reasonable names for these quantities would be df and $d2f$ or ddf .

- (c) plot the graph of $y = f''(x)$ on the interval $-5 < x < 5$ and estimate the intervals on which $y = f(x)$ is concave up.

- (d) create a single plot containing the graphs of $y = f(x)$ and $y = f'(x)$ for $-5 < x < 5$ and verify that $f'(x) = 0$ at each critical number of $y = f(x)$. Did you estimate correctly in (c)?

- (e) Right-click on your graph to see the context menu. Under the option **Legend**, click **Show Legend**. Now, from the same context menu, click **Edit Legend**. Enter an appropriate label for the first curve. Repeat this step for the other curve in the plot.

Assignment

Use Maple to do the following exercises from the textbook: 55 on page 266; 17 and 19 on page 437; 97 and 99 on page 441.