Numerical Integration; Simpson's Rule

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Overview

If it is necessary to evaluate a definite integral of a function for which an antiderivative cannot be found, then one must settle for some kind of numerical approximation of the integral. In Calculus I, we explored Riemann Sum approximations. In this lab, we improve our approximations using Trapezoidal, Midpoint and Simpson's Rule approximations.

Maple Essentials

New Maple commands introduced in this lab include:

Command	Description
ApproximateInt	approximates a definite integral using one of the approximation methods: midpoint, trapezoid or simpson. `ApproximateInt(f(x), x=a..b, method=method, partition=n);` Note: The `ApproximateInt` command is in the `Student[Calculus1]` package.

Notation

Note that the value of `n` when using the Maple command `ApproximateInt` is the same as the value n used in Section 8.7 of your book. Notice that the notation for the Simpson's Rule approximation of the definite integral is S_{2n}. This means that if we want to find S_{10}, we would input `partition=5` since $2n = 10$ implies $n = 5$.

Preparation

Review left- and right-hand Riemann sum approximations for the definite integral. Read Section 8.7 in Anton, Bivens, Davis, 8th edition.

Assignment

1. For the Mastery Quiz you will need to use the various approximation methods to estimate definite integrals and estimate the absolute error that results when using each method. The deadline for turning in Mastery Quiz 6 will be announced in the lab.

2. Next week's lab (Lab H) will be a one-hour, in class quiz. The questions on the quiz will test your ability to apply the information, methods, and techniques in the Maple labs since Hour Quiz 1.

Activities

Use $n = 10$ subintervals to approximate the integral by (a) the midpoint approximation, (b) the trapezoidal approximation, and use $2n = 10$ subintervals to approximate the integral by (c) Simpson's rule. In each case, use the exact value of the integral (as determined by Maple) to calculate the absolute error. Express your answers to at least four decimal places.

1. $\displaystyle\int_0^1 e^{-x^2}dx$

2. $\displaystyle\int_0^2 \frac{x}{\sqrt{1+x^2}}dx$

3. $\displaystyle\int_1^2 \sqrt{x^3 - 1}dx$

4. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{1}{2 - \cos x}dx$

5. $\displaystyle\int_0^2 \sin(x^2)dx$

6. $\displaystyle\int_1^3 \sqrt{\ln x}dx$

Note: These problems appear on pages 566-567 in Anton, Bivens, Davis, 8th Edition.

Example: Activity 1

- Start by including the `Student[Calculus1]` package.
  >with(Student[Calculus1]):

- We define our integrand so we do not have to type as much later.
  >f:=exp(-x^2);

- For (a) we find the midpoint approximation.
  >Mid:=ApproximateInt(f, x=0..1, method=midpoint, partition=10);
  >evalf(Mid);

- We then find the absolute error associated with this approximation.
  >ErrMid:=abs(int(f, x=0..1) - Mid);
  >evalf(ErrMid);

- For (b) we find the trapezoidal approximation.
  >Trap:=ApproximateInt(f, x=0..1, method=trapezoid, partition=10);
  >evalf(Trap);

- We then find the absolute error associated with this approximation.
  >ErrTrap:=abs(int(f, x=0..1) - Trap);
  >evalf(ErrTrap);

- For (c) we find the Simpson's Rule approximation. Notice that $n = 5$ in this case.
  >Simp:=ApproximateInt(f, x=0..1, method=simpson, partition=5);
  >evalf(Simp);

- We then find the absolute error associated with this approximation.
  >ErrSimp:=abs(int(f, x=0..1) - Simp);
  >evalf(ErrSimp);