Piecewise-Defined Functions

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Overview

A skydiver's height above ground is given by different formulae during the free-fall, the opening of the parachute, and the final descent. Mathematically, the height could be written as a single *piecewise-defined function.* The `piecewise` command for working with piecewise-defined functions is introduced in this lab. This will be helpful as you design a goblet.

Maple Essentials

- New Maple commands introduced in this lab include:

Command	Description
`convert`	converts an expression from one form to another form To convert an expression into a piecewise-defined form use: `convert(` f`, piecewise,` x `);`
`piecewise`	define a piecewise-defined function The general syntax to represent $\begin{cases} f_1, & cond_1 \\ f_2, & cond_2 \\ \vdots & \vdots \\ f_n, & cond_n \end{cases}$ is: `piecewise(` $cond_1$`,` f_1`,` $cond_2$`,` f_2`, ...,` $cond_n$`,` f_n `);` where each $cond_i$ is an inequality and each f_i is an expression. It is important to realize that Maple evaluates each $cond_i$ in order. If $cond_j$ is the first condition found to be `true`, the corresponding expression, f_j, is returned. We do not need to write double inequalities as such, only write the $<$ (or $\leq$) part of the double inequality.

Preparation

Recall how to use the `VolumeOfRevolution` command to produce 3-D pictures of solids of revolution and definite integrals for their volume. Recall, from Calculus I, that a function, f, is *continuous at* $x = c$ exactly when $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$.

Assignment

1. Project 1 is due at the beginning of next week's lab. Remember to follow the Project Report Guidelines that are handed out today (and available on the lab homepage). Also, e-mail the Maple worksheet that creates your goblet to your lab TA.
2. For Mastery Quiz 5 you will be asked to write some expressions in the form of piecewise-defined functions.

Activities

1. Consider the function $G(x) = \left|x^2 - 4x\right|$. Use `diff` and `convert` to express the derivative of this function as a piecewise-defined function. Graph $y = G(x)$ and $y = G'(x)$ on the same set of axes. Are there any points where this function is not differentiable?
2. Plot the solid of revolution formed when the region bounded by the graph of $y = G(x)$, from Activity 1, the x-axis, $x = -1/2$, and $x = 3$ is rotated around the x-axis. Notice that this solid, is the shell of a (sideways) goblet.
3. A martini glass is produced when the region bounded by the graphs of $y = F(x) = \begin{cases} 0.1 - 6x, & x < 0 \\ 0.1, & 0 \le x < 7 \\ 2x - 13.9, & x \ge 7 \end{cases}$,

   $y = G(x) = \begin{cases} 0, & x < 7 \\ 2x - 14, & x \ge 7 \end{cases}$, $x = -1/3$ and $x = 9$ is revolved around the x-axis.

   (a) Plot the region and the solid.
   >with(Student[Calculus1]);
   >F:=piecewise(x<0, 0.1-6*x, x<7, 0.1, x>=7, 2*x-13.9);
   >G:=piecewise(x<7, 0, x>=7, 2*x-14);
   >plot([F,G],x=-1/3..9);
   >VolumeOfRevolution(F, G, x=-1/3..9, output=plot);

   (b) How much liquid will this goblet hold?
   >q1:= VolumeOfRevolution(G, 0, x=-1/3..9, output=integral);
   >q1:=value(q1);
   >evalf(q1);

   (c) How much glass is required to make this goblet?
   >q2:=VolumeOfRevolution(F, G, x=-1/3..9, output=integral);
   >q2:=value(q2);

   (d) What is the minimum thickness of glass in this goblet?
   >convert(F-G, piecewise, x);
   The minimum of this function is the minimum thickness of the glass.

   (e) Let R denote the radius of the base of the goblet. The height of the center of mass is located on the x-axis at $x = H$ where $H = \dfrac{\int_a^b (x-a)(f(x)^2 - g(x)^2)\ dx}{\int_a^b (f(x)^2 - g(x)^2)\ dx}$. Compute R, H, and $\frac{H}{R}$.
   > R := eval(F, x=-1/3);
   > H := int((x+1/3)*(F^2-G^2), x=-1/3..9) / int(F^2-G^2, x=-1/3..9);
   > H/R;
   According to the constraint in the project, this would be a very stable goblet. However, the goblet does not hold enough, the glass is too thin, and the region is composed only of linear functions.