Review of Calculus I

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Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review these basic operations and to introduce you to Maple commands and tools for computing limits, derivatives, and integrals.

Maple Essentials

- The *DerivativeDrill*, *AntiDerivativeDrill*, and *IndefiniteIntegralDrill* maplets are useful tools to help you practice your differentiation and integration skills. They are also useful to check any derivative or indefinite integral that you compute by hand — that is, to check your homework answers. These maplets are available from USC on the course website: `http://www.math.sc.edu/calclab/142L-F05/`

- New Maple commands introduced in this lab include:

Command	Description
`diff`	compute derivatives of an expression: `diff( y, x );` finds the derivative of y with respect to x, $\frac{dy}{dx}$; `diff( y, x, x );` finds the second derivative, $\frac{d^2y}{dx^2}$
`int`	used for definite and indefinite integrals: `int( f, x );` evaluates an indefinite integral, $\int f\ dx$; `int( f, x=a..b );` evaluates the definite integral, $\int_a^b f\ dx$.
`limit`	finds one- and two-sided limits: `limit( F, x=a );` returns the two-sided limit, $\lim_{x \to a} F(x)$; `limit( F, x=a, right );` the one-sided limit from above, $\lim_{x \to a^+} F(x)$; `limit( F, x=a, left );` the one-sided limit from below, $\lim_{x \to a^-} F(x)$.
`plot`	plot one or more functions on a specified window `plot( f, x=a..b );` plots the graph of $y = f(x)$ for $a < x < b$; `plot( [f, g], x=a..b );` graphs two functions in a single plot
`:=`	assign a name to a quantity

Note that Maple's **int** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Assignment

This week's Mastery Quiz asks you to use Maple to evaluate some limits, derivatives, and integrals from Exercises in the text. The Activities on the back of this page will help prepare you to answer the Mastery Quiz questions. The deadline for turning in Mastery Quiz 2 will be announced in lab.

Activities

1. Use the *DerivativeDrill* and *AntiDerivativeDrill* maplets to practice your differentiation and integration skills, respectively.

   Hint: *Use the pull-down menus to change the types of functions and rules to be practiced.*

2. Use the *DerivativeDrill* maplet to compute the derivative of the following functions:

   (i) $x^3 \ln(x)$ (ii) $\dfrac{\arctan(x)}{\sqrt{x^2+1}}$ (iii) $\dfrac{1-\ln(t)}{1+\ln(t)}$

3. In a Maple worksheet, evaluate the following indefinite and definite integrals:

   (i) $\displaystyle\int \frac{x+1}{(x-2)^2}\, dx$ (ii) $\displaystyle\int_{-1}^{3} xe^{-x^2}\, dx$ (iii) $\displaystyle\int_{0}^{\pi/2} \left| \frac{1}{2} - \cos x \right|\, dx$

4. In a Maple worksheet, evaluate the following limits:

   (i) $\displaystyle\lim_{x\to 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2}$ (ii) $\displaystyle\lim_{n\to\infty} (1+3/n)^{2n}$ (iii) $\displaystyle\lim_{t\to 1^+} \tan\left(\frac{\pi}{2}t\right)$

5. Let

   $$f(x) = \ln\left(\frac{(x^2+2)^2}{\sqrt{4x^2+1}(x^2+1)} \right).$$

   In a Maple worksheet:

   (a) enter the function and assign it to a name of your choice

                                          Hint: *Use a short, but meaningful, name, say* `f`*.*

   (b) compute the first and second derivatives of $f(x)$

                          Hint: *Reasonable names for these quantities would be* `df` *and* `ddf`*.*

   (c) plot the graph of $y = f''(x)$ on the interval $-5 < x < 5$.

                              Use the plot to estimate the intervals on which f is concave up.

   (d) create a single plot containing the graphs of $y = f(x)$ and $y = f'(x)$ for $-5 < x < 5$.

                              Verify that $f'(x) = 0$ at each critical number of $y = f(x)$.

Additional Notes

- The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Symbol** palette contains other symbols, including π (`Pi`) and ∞ (`infinity`). To view both palettes simultaneously, drag one of the palettes to another edge of the Maple window.

- Listen carefully when your lab instructor explains how to use the right mouse button to perform operations on an expression.