New Functions from Old

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Overview

In this lab, we will use Maple to help us calculate and simplify combinations of functions. The **Shift** maplet will also be introduced to help us practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- The *Shift* maplet is available from the course website:

    http://www.math.sc.edu/calclab/141L-S09/labs/ → <u>Shift</u>

- New Maple commands introduced in this lab include:

Command	Description
`simplify`	`simplify(f(x));` simplifies $f(x)$

Preparation

Read Section 1.3: *New Functions from Old* in Anton.

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.

    (a) From the Calculus I Lab Assignments page under Lab C, click on <u>Shift</u>. You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password.

    (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.

    (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

    Note: *If you do not get the answer correct, the graph of your equation will be displayed in red.*

2. In each of the problems on the next page, you will use the assignment operator (`:=`) together with the arrow notation (`->`) to define each function. Once you have done this, the problems are straightforward. We will work through one example together.
    Note: *Remember that you can use the Expression and Common Symbols palettes to avoid typing so much. You may also find the labels useful.*

Example Problems

For the following problems, let $f(x) = \dfrac{1}{x^2 - 4}$ and $g(x) = \dfrac{1}{x - 2}$.

    a. Find and simplify formulas for $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$.

    b. Evaluate the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$. Simplify your answer.

    c. Find and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$.

    d. Express f as a composition of two functions: $f(x) = (f1 \circ f2)(x)$.

We begin by defining the functions using proper Maple notation.
```
> f:=x-> 1/(x^2-4);
> g:=x-> 1/(x-2);
```
We can then evaluate each of the operations on the functions.
```
> f(x)+g(x);          To simplify:
> f(x)-g(x);          Right-click over the expression.
> f(x)*g(x);          Choose Simplify from the context menu.
> f(x)/g(x);
```
For the difference quotient, remember that you will need parentheses around the numerator.
```
> (f(x+h)-f(x))/h;
```
Evaluate the compositions as follows. Remember to simplify.
```
> f(g(x));
> g(f(x));
```
For the decomposition of $f(x)$, you will have to do some work on your own:
```
> f1:=x-> ??;
> f2:=x-> ??;
> f1(f2(x));
```
Did you get $f(x)$?

Additional Practice

    1. Find and simplify formulas for $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$.
        a. $f(x) = 2\sqrt{x - 1}$, $g(x) = \sqrt{x - 1}$ (Ex. 29 on Page 37)
        b. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$

    2. Let $f(x) = x^2 + 1$. Find and simplify each of the following. (Ex. 34 on Page 37)
        a. $f(t + 2)$ b. $f(\sqrt{x})$ c. $f(f(x))$

    3. Evaluate the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$. Simplify your answer.
        a. $f(x) = 3x^2 - 5$ (Ex. 53 on Page 38)   b. $f(x) = \frac{1}{(x+1)^2}$

    4. Find and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$.
        a. $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$ (Ex. 37 on Page 37)
        b. $f(x) = \sqrt{2x + 3}$, $g(x) = x^2 + 1$

    5. Express F as a composition of two functions: $F(x) = (f \circ g)(x)$. (Ex. 43 on Page 37)
        a. $F(x) = \sin^2 x$ b. $F(x) = \frac{3}{5+\cos x}$ c. $F(x) = (x^2 + 1)^{10}$

Assignment

With the help of Maple, work out exercises 12, 30, 32, and 56 of §1.3 (pages 36-38). Write your answers on a sheet of paper to be turned in with your Maple worksheet.