

New Functions from Old

Douglas Meade, Ronda Sanders, and Xian Wu
Department of Mathematics

Overview

In this lab, we will use Maple to help us calculate and simplify combinations of functions. The **Shift** maplet will also be introduced to help us practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- The *Shift* maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S08/labs/> → [Shift](#)

- New Maple commands introduced in this lab include:

Command	Description
<code>simplify</code>	<code>simplify(f(x))</code> ; simplifies $f(x)$

Preparation

Read Section 1.3: *New Functions from Old* in Anton.

Activities

- Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
 - From the Calculus I Lab Assignments page under Lab C, click on [Shift](#). You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password.
 - This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
 - To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

Note: If you do not get the answer correct, the graph of your equation will be displayed in red.

- In each of the problems on the next page, you will use the assignment operator (`:=`) together with the arrow notation (`->`) to define each function. Once you have done this, the problems are straightforward.

Note: Remember that you can use the *Expression and Common Symbols* palettes to avoid typing so much. You may also find the labels useful.

- Find and simplify formulas for $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$.
 - a. $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$ (Ex. 29 on Page 37)
 - b. $f(x) = 1 + \frac{x}{x+1}$, $g(x) = 2 - \frac{1}{x}$
- Let $f(x) = x^2 + 1$. Find and simplify each of the following. (Ex. 34 on Page 37)
 - a. $f(t+2)$
 - b. $f(\sqrt{x})$
 - c. $f(f(x))$
- Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$. Simplify your answer.
 - a. $f(x) = 3x^2 - 5$ (Ex. 53 on Page 38)
 - b. $f(x) = \frac{1}{(x+1)^2}$
- Find and simplify $(f \circ g)(x)$ and $(g \circ f)(x)$.
 - a. $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$ (Ex. 37 on Page 37)
 - b. $f(x) = \sqrt{2x+3}$, $g(x) = x^2 + 1$
- Find and simplify $(f \circ g \circ h)(x)$.
 - a. $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 2$, $h(x) = \cos x$
 - b. $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, $h(x) = x^3$ (Ex. 39 on Page 37)
- Express F as a composition of two functions; that is, find f and g such that $F = (f \circ g)(x)$. Use Maple to verify the composition. (Ex. 43 on Page 37)

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = \sin^2 x$
 - b. $F(x) = \frac{3}{5+\cos x}$
 - c. $F(x) = (x^2 + 1)^{10}$
- Express F as a composition of three functions; that is, find f , g , and h such that $F = (f \circ g \circ h)(x)$. Use Maple to verify the composition. (Ex. 45 on Page 37)

Note: Do not choose the identity ($y = x$) as one of your functions.

 - a. $F(x) = (1 + \sin(x^2))^3$
 - b. $F(x) = \sqrt{1 - x^{1/3}}$
 - c. $F(x) = \cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ if $f(x) = \frac{4}{3+x^2}$. Simplify your answer.


```
> f:= x -> 4 / (3+x^2);
> (f(x+h) - f(x)) / h;
> simplify(label);
```

Note: You should right-click over your expression and choose an action. If you choose to type the command, use **Ctrl-L** to insert a label.
2. Find and simplify $(f \circ g \circ h)(x)$ if $f(x) = \frac{2}{1-x^2}$, $g(x) = \sin(x)$, and $h(x) = \sqrt{x}$.


```
> f:= x -> 2 / (1-x^2);
> g:= x -> sin(x);
> h:= x -> sqrt(x);
> f(g(h(x)));
> simplify(label);
```

Assignment

With the help of Maple, work out exercises 12, 30, 32, and 56 of §1.3 (pages 36-38). Write your answers on a sheet of paper to be turned in with your Maple worksheet.