

# Differentiation and Tangent Lines

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## Overview

This week's lab will provide practice finding locally linear approximations to functions. That is, we will be finding the equation of the tangent line to a curve.

## Maple Essentials

- The *Tangents* tutor is started from the Maple 11 user interface under the Tools menu:

**Tools** → **Tutors** → **Calculus - Single Variable** → **Tangents...**

- The *TangentLine* maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S08/labs/> → [TangentLine](#)

- The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

Command	Description
<code>:=</code>	assignment
<code>x -&gt;</code>	function definition
<code>plot</code>	plot one or more expressions

## Preparation

Review §3.1 and §3.2 in Anton. Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1)$$

where  $(x_1, y_1)$  is a point on the line and  $m$  is the slope of the line. Next, solve the equation for  $y$  and we get:

$$y = m(x - x_1) + y_1.$$

Now, we use the substitution  $y_1 = f(x_1)$  and this becomes:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at  $x_1$  is the same as the slope of the tangent line at  $x_1$ . Thus we get the following formula for the equation of the tangent line at  $x_1$ :

$$y = f'(x_1)(x - x_1) + f(x_1).$$

## Activities

We will find the equation of the tangent line to the graph of  $f(x)$  at the point  $(x_1, f(x_1))$  for several different functions. We will then graph the function and its tangent line on the same axes.

**Example Problem**

We will solve the following problem together in two different ways:

- Find an equation for the line that is tangent to the graph of the differentiable function  $f(x) = x^3 - 2x + 1$  at  $x_1 = 2$ . Then graph the curve and this tangent line on the same axes.

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as  $x^3-2*x+1$  and  $x=2$ , and change the number of iterations to 5.
3. Click **Display**. The tutor will display the function and a series of secant lines, including the tangent line. The equation of the tangent line is displayed on the right.
4. Press the **Animate** button. The tutor will show the progression through the secant lines as  $\Delta x$  gets smaller.
5. The tutor will return the last graph when you click **Close**.
6. If you want to graph the function and the tangent line, assign both in a Maple worksheet and write a plot command.

The second way:

1. Define and assign the function to  $f$ .  

$$> f := x \rightarrow x^3-2*x+1;$$
2. Right-click and choose *Differentiate*. Then use a label (or right-click again) to assign this new function to  $df$ .  

$$> df := label;$$
3. Find  $f'(2)$  and assign that value to  $m$ .  

$$> m := df(2);$$
4. Find the tangent line  $y = f'(2)(x - 2) + f(2)$  and assign it to  $L$ .  

$$> L := m*( x-2 ) + f(2);$$
5. Plot the function and the tangent line using different linestyles.  

$$> plot( [f(x), L], x=-2..3, linestyle=[solid, dash]);$$

**Functions**

Find the equation of the tangent line to the graph of  $f(x)$  at the point  $(x_1, f(x_1))$ . Graph the function and its tangent line on the same axes. Choose a window that will give you a good view of both the function and the tangent line.

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|--|---|--|
| a. $f(x) = \sqrt{x}$ , $x_1 = \frac{1}{4}$ | b. $f(x) = \frac{5}{x} + 1$ , $x_1 = -2$    | c. $f(x) = x^2$ , $x_1 = 1$            |
| d. $f(x) = 2^x$ , $x_1 = 3$                | e. $f(x) = \cos(x)$ , $x_1 = \frac{\pi}{6}$ | f. $f(x) = \frac{1}{5x-3}$ , $x_1 = 1$ |

**Additional Practice**

The *TangentLine* maplet gives a step by step method for finding the equation of the tangent line to  $f(x)$  at the point  $x = a$ . Launch the maplet and click **New Function**. Follow the prompts to find  $f(a)$ ,  $f'(x)$ , and  $f'(a)$ . Then enter the equation of the tangent line as follows:

$$y = f'(a)(x - a) + f(a).$$

The maplet will check each of your answers and let you know whether you are correct.

**Assignment**

Exercises 49, 59, and 60 of §3.6 on pages 214-215. Give each of your graphs a title and a legend.