New Functions from Old

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Overview

There are three objectives in this lab:

- Learn to input and manipulate functions using Maple 10.
- Use Maple 10 to calculate and simplify combinations of functions.
- Learn to use Maplets for Calculus.

Maple Essentials

- The *Shift* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-S07/labs/` → Shift

- New Maple commands introduced in this lab include:

Command	Description
`->`	`f:= x -> a*x+b;` assigns f to be the function $f(x) = ax + b$
`simplify`	`simplify(F);` simplifies expression F

Preparation

Read Section 1.3: *New Functions from Old* in Anton.

Assignment

This week's Mastery Quiz asks you to use Maple to evaluate (and simplify) combinations and compositions of functions as well as identify the graph of a shifted function. The Activities in this lab will help prepare you to answer the Mastery Quiz questions. The deadline for turning in Mastery Quiz 2 will be announced in lab.

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
   (a) From the Calculus I Lab Assignments page under Lab C, click on Shift. You will be prompted for a username and password as these Maplets are protected. You should use your Blackboard username and password. (If you have never used Blackboard, use your social security number as your password.)
   (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
   (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

   Note: *If you do not get the answer correct, the graph of your equation will be displayed in red.*

2. In each of the following problems, you will use the arrow notation (`->`) to define each function and the assignment operator (`:=`) to assign each function to a name. Once you have done this, the problems are straightforward.

   Note: *Remember that you can use the Expression and Common Symbols palettes to avoid typing so much. You may also find the labels useful.*

   - Find and simplify formulas for $f+g$, $f-g$, fg, and f/g.
     a. $f(x)=\frac{x}{x-1},\ g(x)=\frac{1}{x}$
     b. $f(x)=\frac{2}{x+1},\ g(x)=\frac{x}{x+1}$
     c. $f(x)=1+\frac{x}{x+1},\ g(x)=2-\frac{1}{x}$
   - Let $g(x)=4-\frac{1}{3x+2}$. Find and simplify each of the following.
     a. $g(5s+2)$ b. $3g(5x)$ c. $g(g(x))$
   - Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h},\ h\neq 0$. Simplify your answer.
     a. $f(x)=3x^2-x+7$ b. $f(x)=\frac{1}{(x+1)^2}$
   - Find and simplify $(f\circ g)(x)$ and $(g\circ f)(x)$.
     a. $f(x)=x+\frac{1}{x},\ g(x)=\frac{x+1}{x+2}$
     b. $f(x)=\sqrt{2x+3},\ g(x)=x^2+1$
   - Find and simplify $(f\circ g\circ h)(x)$.
     a. $f(x)=\sqrt{x-1},\ g(x)=x^2+2,\ h(x)=x+3$
     b. $f(x)=\frac{2}{x+1},\ g(x)=\cos x,\ h(x)=\sqrt{x+3}$
   - Express F as a composition of two functions; that is, find f and g such that $F=f\circ g$. Use Maple to verify the composition.
     Note: *Do not choose the identity ($y=x$) as a function.*
     a. $F(x)=(x^2+1)^{10}$ b. $F(x)=\sin(\sqrt{x})$ c. $F(x)=\frac{\tan x}{1+\tan x}$
   - Express F as a composition of three functions; that is, find f, g, and h such that $F=f\circ g\circ h$. Use Maple to verify the composition.
     Note: *Do not choose the identity ($y=x$) as a function.*
     a. $F(x)=1-3^{x^2}$ b. $F(x)=\sqrt{2+|x|}$ c. $F(x)=\cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h},\ h\neq 0$ if $f(x)=\frac{4}{3+x^2}$. Simplify your answer.

```
> f:= x -> 4 / (3+x^2);
> (f(x+h) - f(x)) / h;
> simplify(label);
```

2. Find and simplify $(f\circ g\circ h)(x)$ if $f(x)=\frac{2}{1-x^2}$, $g(x)=\sin(x)$, and $h(x)=\sqrt{x}$.

```
> f:= x -> 2 / (1-x^2);
> g:= x -> sin(x);
> h:= x -> sqrt(x);
> f(g(h(x)));
> simplify(label);
```

Note: *Use* **ctrl-L** *to insert a label.*