A More Rigorous Approach to Limits

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Overview

The rigorous ϵ–δ definition of limits can be difficult for students to grasp. This lab is designed to provide visual and interactive tools for working with these concepts.

Maple Essentials

- The *EpsilonDelta* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-S07/labs/` → EpsilonDelta

Preparation

Review the precise definition of the limit (pages 138–142 in Anton).
DEFINITION: Let $f(x)$ be defined for all x in some interval containing the number a, with the possible exception that $f(x)$ need not be defined at a. We will write

$$\lim_{x \to a} f(x) = L$$

if given any number $\epsilon > 0$ we can find a number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ if } 0 < |x - a| < \delta.$$

Maple Syntax

For precise solutions to our inequalities, we will be using Maple's `solve` command. The general syntax is
\> solve(eqn, var);
where *eqn* is the equation (or inequality) and *var* is the variable for which we want to solve. We will input most of our inequalities as follows
\> solve(abs(f(x)−L) < ϵ, x);
For example, if we want to know where $|\sqrt{x} - 2| < 0.05$ we would use the following command
\> solve(abs(sqrt(x)−2) < 0.05, x);
and Maple would return the interval $(3.8025, 4.2025)$.

Assignment

This week's Mastery Quiz will be very similar to the Activities in this lab. **Next week is Hour Quiz 1 – to be completed in lab.** As preparation, you should review the information and Maple commands in Labs A-E.

Activities

When using the $\epsilon - \delta$ definition of the limit, we want to find the largest δ that satisfies the definition. For each of the limits below, your task is to identify the δ for each ϵ given.
(Follow the General Directions at the bottom of the page.)

1. $\lim_{x \to 9} \sqrt{x} = 3,\ \epsilon = 0.15,\ \epsilon = 0.05$
2. $\lim_{x \to 3} (4x - 5) = 7,\ \epsilon = 0.4,\ \epsilon = 0.2$
3. $\lim_{x \to 3} (5x - 2) = 13,\ \epsilon = .05,\ \epsilon = .01$
4. $\lim_{x \to 2} (x^2 + 3x - 1) = 9,\ \epsilon = 0.8,\ \epsilon = 0.6$
   HINT: For this one, you should use the interval that contains a.

General Directions

1. Look at the limit and identify $f(x)$, L, a, and ϵ.
2. Launch the *EpsilonDelta* maplet.
3. Enter the function $f(x)$, a, and L. Enter ϵ.
4. Click on the **Plot Setup** button. Enter an appropriate viewing window. Remember, your x-range should include a and your y-range should include L.
5. Click **Accept**. You should see the graph of $f(x)$ in blue with pink shading that goes from $x - \delta$ to $x + \delta$ along the x-axis and brown shading that goes from $f(x - \delta)$ to $f(x + \delta)$ along the y-axis. You will notice two red horizontal lines, one at $L - \epsilon$ and the other at $L + \epsilon$.
6. Your task is to increase the value of δ as far as possible without shading beyond the red lines. You should zoom several times to insure that you have not crossed either horizontal line.
7. When you think you are done, write down your last value of δ that did not cross the line.
8. Now we will find the value of δ more precisely.
9. Use the arrow notation (`->`) to assign the function $f(x)$. Use `:=` to assign L, a, and `epsilon` to their respective values.
10. Use the `solve` command as follows
    > solve(abs(f(x) − L) < epsilon, x);
    Maple will return an interval.
11. Find the distances from a to the left bound and from a to the right bound of the interval. (Remember you should use absolute value so both distances are positive.) The *smallest* of these two values is the *largest* δ that works for this ϵ.
12. Your values from the *EpsilonDelta* maplet and from using the `solve` command should be very close.