

Limits, Infinity, and Asymptotes

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Overview

Asymptotes for functions are sometimes easy to identify from a graph. The actual definitions of asymptotes is in terms of limits:

Asymptote	Equation	Definition
Horizontal	$y = L$	$\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$
Vertical	$x = a$	$\lim_{x \rightarrow a^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow a^-} f(x) = \pm\infty$

This lab is designed to provide experience finding asymptotes. Some limits will be evaluated by inspection. You will learn several ways to use Maple to help evaluate more difficult limits.

Maple Essentials

- New Maple commands introduced in this lab include:

Command	Description
<code>limit</code>	finds one- and two-sided limits: <code>limit(F, x=a)</code> ; returns the two-sided limit, $\lim_{x \rightarrow a} F(x)$ <code>limit(F, x=a, right)</code> ; the one-sided limit from above, $\lim_{x \rightarrow a^+} F(x)$ <code>limit(F, x=a, left)</code> ; the one-sided limit from below, $\lim_{x \rightarrow a^-} F(x)$
<code>factor</code>	<code>factor(F)</code> ; factors expression F
<code>solve</code>	<code>solve(F > 0)</code> ; returns the real range over which $F > 0$

- The *Rational Functions* tutor is started from the Maple 9.5 user interface under the Tools menu:

Tools → **Tutors** → **Precalculus** → **Rational Functions** ...

- The *LimitCheck* maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S06/labs/> → [LimitCheck](#)

Preparation

To prepare for this lab, review the definitions of horizontal and vertical asymptotes. Also review basic facts about rational and exponential functions.

Assignment

This week's Mastery Quiz asks you to use Maple to identify horizontal and vertical asymptotes for several functions. The Activities in this lab will help prepare you to answer the Mastery Quiz questions. The deadline for turning in Mastery Quiz 4 will be announced in lab.

Activities

Your task is to identify all horizontal and vertical asymptotes for as many of the functions on the back of this page as possible. For [rational functions](#) the *Rational Functions* tutor can be used to obtain a graph of the function and its asymptotes. This is a start, but you still need to identify the exact equations for the asymptotes.

Rational Functions: General Directions

- Look at the function $f(x)$ and determine which values make the denominator zero. (Use the command `factor(F)`; for factoring if necessary.) These values will be the a 's that we need to check as possible vertical asymptotes.
- Define f as your function and a as one of the values to be checked using `:=` as previously discussed.
- Enter the following lines of code:
 - `limit(f, x=a, left);`
 - `limit(f, x=a, right);`
 If either of these returns the value ∞ or $-\infty$ then $x = a$ is the equation of a vertical asymptote of $f(x)$.
- Enter the following lines of code:
 - `limit(f, x=infinity);`
 - `limit(f, x=-infinity);`
 If either of these returns a value $L \neq \pm\infty$ then $y = L$ is the equation of a horizontal asymptote of $f(x)$.
- Use the *Rational Functions* tutor to get a graph of each function.
Note: The *Rational Functions* tutor does not provide any control over the viewing window. However, when you click **Close** the plot is returned to the Maple worksheet. From here the viewing window can be modified via the **Axes** entry in the context menu produced when the right mouse button is pressed when the cursor is over the graph.

Rational Functions

$$\begin{array}{lll}
 \text{(i) } f(x) = \frac{3x^2+2x-1}{x+2} & \text{(ii) } f(x) = \frac{5+2x}{1+x} & \text{(iii) } f(x) = \frac{x^3+3x^2-12x+4}{x^3-4x} \\
 \text{(iv) } f(x) = \frac{3x^2+1}{x^2+2x-15} & \text{(v) } f(x) = \frac{2x^2-x-1}{x^3-2x^2-x+2} & \text{(vi) } f(x) = \frac{x^2-1}{5x^2+1}
 \end{array}$$

Exponential Functions: General Directions

- Recall in the definition of an exponential function of the form $y = b^x$, we have the stipulation that $b > 0$. We will use this information to determine values a that are potential vertical asymptotes.
- Use the `solve` command to find the real range over which the base of the exponential is positive. The endpoints of your intervals are the a 's we need to check. (You should choose left or right based on the interval. That is, test the limit from the direction where the base is positive.)
- Follow steps 2-4 in the *Rational Functions: General Directions* section above.
- Use the given window to plot the graph of each function.

Exponential Functions

$$\begin{array}{lll}
 \text{(i) } f(x) = \left(1 + \frac{3}{x}\right)^x & \text{(ii) } f(x) = \left(\frac{5+x}{x}\right)^x & \text{(iii) } f(x) = \left(\frac{x+2}{x+3}\right)^x \\
 [-50,50] \times [-100,100] & [-100,100] \times [-1000,1000] & [-10,10] \times [-5,5]
 \end{array}$$

Other Functions

What makes these functions different?

$$\text{(i) } f(x) = \frac{\sqrt{x^2+4}-2}{x} \quad \text{(ii) } f(x) = \frac{\sqrt{x^2+1}+2x}{x} \quad \text{(iii) } f(x) = \frac{\sin x}{x}$$