Implicit Differentiation

Douglas Meade and Ronda Sanders
Department of Mathematics

Overview

This lab provides experience working with functions defined implicitly. The first task is to be able to graph an implicitly-defined function. Next, the equation of the tangent line at a point on the graph of an implicitly-defined function is found and added to the graph.

Maple Essentials

- The *ImplicitDifferentiation* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-S06/labs/` → <u>ImplicitDifferentiation</u>

- The new Maple commands introduced in this lab are:

Command	Description
`display`	combine one or more plots in a single plot; part of the `plots` package
`implicitdiff`	compute derivatives for implicitly-defined functions
`implicitplot`	create graph of function defined implicitly; part of the `plots` package
`pointplot`	plots a single point; part of the `plots` package
`with`	loads the contents of a Maple package

Preparation

Review implicitly-defined functions and implicit differentiation (pp. 235–241 in Anton). Also, review the methods for finding and plotting tangent lines (for functions defined explicitly).

Assignment

This week's mastery quiz asks you to find and plot the tangent line to an implicitly-defined curve. The activities on today's lab will help you answer the mastery quiz questions.

Activities

We will find the equation of the tangent line to the graph of an implicitly-defined function at the point (x_1, y_1) for several functions. We will then graph the curve, the point, and the tangent line.

Example Problem

We will solve the following problem together:

- Find an equation for the line that is tangent to the graph of the implicitly-defined function $y^3 + yx^2 + x^2 - 3y^2 = 0$ at the point (-1,1). Then graph the curve, the point, and the tangent line using a viewing window of [-5,5]×[-2,4].

Steps:

1. First, load the Maple `plots` package. Without the contents of this package, much of what we do today will not work.
   > `with(plots):`
2. Assign our equation using ':='.
   > `eq:= y^3+y*x^2+x^2-3*y^2=0;`
3. Find (and assign) the derivative using implicit differentiation. Since we want $\frac{dy}{dx}$, we input `y` and then `x`.
   > `dydx:= implicitdiff(eq, y, x);`
4. Find (and assign) the slope of the tangent line at the point (-1,1).
   > `m:= eval(dydx, {x=-1, y=1});`
5. Find (and assign) the equation of the tangent line. Remember: $y = m(x - x_1) + y_1$.
   > `L:= m*(x+1)+1;`
6. Next, write (and assign) commands to plot the curve, the point, and the tangent line. Write the commands separately using ':' so Maple does not display the output yet. (In the first plot command, the option `numpoints=10000` will insure a smooth curve.)
   > `P1:= implicitplot(eq, x=-5..5, y=-2..4, numpoints=10000):`
   > `P2:= pointplot([-1,1], color=green, symbolsize=15):`
   > `P3:= plot(L, x=-5..5, y=-2..4, color=blue, linestyle=DOT):`
7. Use the `display` command to display the curve, point, and tangent line on a single plot.
   > `display([P1, P2, P3], title=''Figure 1'');`

Problems

For each of the following implicitly-defined functions, find the equation of the tangent line at the given point (x_1, y_1). Then graph the curve, the point, and the tangent line on a single plot using the given viewing window.

- Equation: $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
  Point: $(3, 1)$
  Viewing Window: [-5,5]×[-4..4]
- Equation: $x^2y - xy^2 = 2$
  Point: $(-1, -2)$
  Viewing Window: [-5,5]×[-5..5]
- Equation: $x^3 + y^3 = 3xy$
  Point: $(\frac{3}{2}, \frac{3}{2})$
  Viewing Window: [-3,3]×[-4..3]

Additional Notes

The ImplicitDifferentiation maplet provides additional practice finding the slope of a curve at a point.