# Differentiation and Tangent Lines

Douglas Meade and Ronda Sanders
Department of Mathematics

## *Overview*

This week's lab will provide practice finding locally linear approximations to functions. That is, we will be finding the equation of the tangent line to a curve.

## *Maple Essentials*

- The *Tangents* tutor is started from the Maple 9.5 user interface under the Tools menu:

  **Tools → Tutors → Calculus - Single Variable → Tangents...**

- The *TangentLine* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-S06/labs/` → TangentLine

- The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

| Command | Description |
|---|---|
| `:=` | assignment |
| `diff` | `diff( F, x )`; differentiate an expression |
| `eval` | evaluate at a point |
| `plot` | plot one or more expressions |

## *Preparation*

Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1)$ is a point on the line and $m$ is the slope of the line. Next, solve the equation for $y$ and we get:

$$y = m(x - x_1) + y_1.$$

Now, we use the substitution $y_1 = f(x_1)$ and this becomes:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at $x_1$ is the same as the slope of the tangent line at $x_1$. Thus we get the following formula for the equation of the tangent line at $x_1$:

$$y = f'(x_1)(x - x_1) + f(x_1).$$

## *Assignment*

This week's mastery quiz asks you to find and graph the tangent lines for several functions. The Activities in this lab will help you answer the Mastery Quiz questions. The *TangentLine* maplet provides additional practice finding tangent lines.

## *Activities*

We will find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$ for several different functions. We will then graph the function and its tangent line on the same axes.

## *Example Problem*

We will solve the following problem together in two different ways:

- Find an equation for the line that is tangent to the graph of the differentiable function $f(x) = x^3 - 2x + 1$ at $x_1 = 2$. Then graph the curve and this tangent line on the same axes.

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as x^3-2*x+1 and the base point as 2.
3. Press the **Display** button.
4. The maplet will return the graph of the function and the tangent line when you press **Close**.

The second way:

1. Enter and execute one by one the following Maple commands.

| | |
|---|---|
| > f := x^3-2*x+1; | Assign the function to f. |
| > Df := diff( f, x ); | Compute the derivative f'(x) and assign to Df. |
| > m := eval( Df, x=2 ); | Find f'(2) and assign to m. |
| > L := m*( x-2 )+eval( f, x=2 ); | Find the tangent line y=f'(2)(x-2)+f(2). |
| > plot( [f, L], x=-2..3 ); | Plot the function and the tangent line. |

## *Functions*

Find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$. Graph the function and its tangent line on the same axes.

1. $f(x) = \sqrt{x}, x_1 = \frac{1}{4}$
2. $f(x) = \frac{5}{x} + 1, x_1 = -2$
3. $f(x) = x^2, x_1 = 1$
4. $f(x) = 2^x, x_1 = 1$
5. $f(x) = \sin(x), x = \frac{\pi}{4}$

## *Additional Notes*

- The `diff` command can also be used to evaluate higher order derivatives. For example, if we wanted to find the second derivative of `F`, we would use the command `diff( F, x, x )`. This notation, however, becomes somewhat tedious for higher order derivatives. The command `x$n` repeats `x` successively `n` times and thus shortens our commands. For example, for the fourth derivative of `F`, we could write `diff( F, x$4 )` instead of `diff(F, x, x, x, x )`.
- **Not using a color printer?** If you want to distinguish the curves on your graph, add the option `linestyle=[SOLID, DOT]` to your plot command. This will draw $f(x)$ with a solid line and the tangent line with dots.