

PROJECT 1: LINEAR FIT TO TANGENT LINES

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Overview

An important problem addressed by calculus is that of finding a good linear approximation to the function $f(x)$ near a particular x -value, say $x = p$. One possible approach (not the best) is to sample values of the function near $x = p$, find the least-squares line for this data, and translate the least-squares line so that it passes through the point $(p, f(p))$, which is on the graph of $y = f(x)$. We can then zoom in to determine the quality of the fit. We will use this method in the following project.

Maple Syntax

In Maple, we get the number π using the notation Pi.

Project

For this project, we will use the function $f(x) = 2 + x + |\sin(x)|$.

- (1) Let $p = 2$. Complete the following steps.
 - (a) Use your calculator to create a table of $(x, f(x))$ values for five equally-spaced x -values on the interval $[p - 0.1, p + 0.1]$. All values should be accurate to at least three digits to the right of the decimal.
 - (b) Find the least-squares line for the data in (a).
 - (c) Find the equation of the line parallel to the least-squares line that passes through the point $(p, f(p))$.
 - (d) Create a figure that displays the graph of $y = f(x)$ and the graph of the line found in (c). You should only show the portion of the graph close to p , say from $p - 0.1$ to $p + 0.1$.
 - (e) Is the graph of the line found in (c) a good linear approximation to the graph of $y = f(x)$ near the point $(p, f(p))$?
- (2) Let $p = 0$. Repeat steps (a)-(e) from above.
- (3) Give a general rule for the values of p for which this method yields a good approximation to the graph of $y = f(x)$ near $x = p$. For this, you should try several other values of p , for example $p = 1$, $p = \frac{\pi}{2}$, $p = 3$, $p = \pi$, $p = -\frac{\pi}{2}$, $p = 2\pi$, etc.
- (4) Create two additional figures: one that shows another p -value that yields a good linear approximation and one that shows another p -value that does not yield a good linear approximation.

Check List

- Does the line in your figure intersect the graph of $f(x)$? It should.
- Was your calculator in Radian mode? It should have been.
- Is your project report neat and clean?
- Does every figure have a title and legend? Can you distinguish the curves from one another?