

## New Functions from Old

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### Overview

In this lab, we will use Maple to help us to calculate and simplify combinations of functions. A maplet (**Shift**) will also be introduced to help us to practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

### Maple Essentials

- Important Maple commands introduced in this lab:

Command	Description	Example
<code>:=</code> <code>&gt;</code>	define a function in $x$	<code>f:=x-&gt;2*x^2*(1-x^2);</code>
<code>simplify</code>	simplify functions/expressions	<code>simplify(f(x));</code> <code>simplify(x-(x-1)^8);</code>
<code>eval</code>	evaluate functions/expressions	<code>eval(f(x),x=2);</code> <code>eval((x-h)^9,h=0);</code>

- The **Shift** maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-F12/labs/> → [Shift](#)

### Related course material/Preparation

Calculus Text: §1.3. Maple Text: §2.1.

### Assignment

Complete lab activities and your lab instructor will give other assignment for each section

### Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
  - (a) From the Calculus I Suggested Lab Schedule/Assignments page under Lab 2, click on [Shift](#). You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password. (You may need to reset your password there: login to your VIP, go to the TECHNOLOGY, choose the second one from the TECHNOLOGY Menu.)
  - (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
  - (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

**Note:** If you do not get the answer correct, the graph of your equation will be displayed in red.

2. In each of the following problems, you will use the assignment operator ( $:=$ ) together with the arrow notation ( $x \rightarrow$ ) to define each function. Once you have done this, the problems are straightforward. Remember that you have to call a function **together with its variable** (like  $f(x)$ ,  $f(t)$ ,  $f(2)$ , or  $f(\text{whatever})$ ). Maple won't recognize a function just by his name (like  $f$ ).

**Note:** You can always use the Expression, Common Symbols, and/or Favorites palettes to avoid typing so much. You may also find the labels useful.

- Find and simplify formulas for  $f(x) + g(x)$ ,  $f(x) - g(x)$ ,  $f(x)g(x)$ , and  $f(x)/g(x)$ .
  - a.  $f(x) = 2\sqrt{x-1}$ ,  $g(x) = \sqrt{x-1}$
  - b.  $f(x) = 1 + \frac{x}{x+1}$ ,  $g(x) = 2 - \frac{1}{x}$
- Let  $f(x) = x^2 + 1$ . Find and simplify each of the following.
  - a.  $f(5s + 2)$
  - b.  $3f(1/x)$
  - c.  $f(f(x))$
- Evaluate  $\frac{f(x+h)-f(x)}{h}$ . Simplify your answer and then let  $h$  go to 0.
  - a.  $f(x) = 3x^2 - 5$
  - b.  $f(x) = \frac{1}{(x+1)^2}$
- Find and simplify compositions  $(f \circ g)(x) = f(g(x))$  and  $(g \circ f)(x) = g(f(x))$ .
  - a.  $f(x) = \frac{1+x}{1-x}$ ,  $g(x) = \frac{x}{1-x}$
  - b.  $f(x) = \sqrt{2x+3}$ ,  $g(x) = x^2 + 1$
- Find and simplify  $(f \circ g \circ h)(x)$ .
  - a.  $f(x) = \sqrt{x-1}$ ,  $g(x) = x^2 + 2$ ,  $h(x) = \cos x$
  - b.  $f(x) = x^2 + 1$ ,  $g(x) = \frac{1}{x}$ ,  $h(x) = x^3$
- Express  $F(x)$  as a composition of two functions; that is, find  $f(x)$  and  $g(x)$  such that  $F(x) = f \circ g(x)$ . Use Maple to verify the composition.
 

**Note:** Do not choose the identity ( $y = x$ ) as one of your functions.

  - a.  $F(x) = \sin^2 x$
  - b.  $F(x) = \frac{3}{5+\cos x}$
  - c.  $F(x) = (x^2 + 1)^{10}$
- Express  $F(x)$  as a composition of three functions; that is, find  $f(x)$ ,  $g(x)$ , and  $h(x)$  such that  $F = f \circ g \circ h(x)$ . Use Maple to verify the composition.
 

**Note:** Do not choose the identity ( $y = x$ ) as one of your functions.

  - a.  $F(x) = (1 + \sin(x^2))^3$
  - b.  $F(x) = \sqrt{1 - x^{1/3}}$
  - c.  $F(x) = \cos^4(\sqrt{x})$

### Example Problems

1. Evaluate the difference quotient  $\frac{f(x+h)-f(x)}{h}$ ,  $h \neq 0$  if  $f(x) = \frac{4}{3+x^2}$ . Simplify your answer and then let  $h$  go to 0.
 

```
> f:= x -> 4 / (3+x^2);
> (f(x+h) - f(x)) / h;
> simplify(label);
> eval(label, [h = 0, x = x]);
```

**Note:** You should right-click over your expression and choose an action. If you choose to type the command, use **Ctrl-L** to insert a label.
2. Find and simplify  $f \circ g \circ h(x)$  if  $f(x) = \frac{2}{1-x^2}$ ,  $g(x) = \sin(x)$ , and  $h(x) = \sqrt{x}$ .
 

```
> f:= x -> 2 / (1-x^2);
> g:= x -> sin(x);
> h:= x -> sqrt(x);
> f(g(h(x)));
> simplify(label);
```