

Limits, Infinity, and Asymptotes

Douglas Meade, Ronda Sanders, and Xian Wu

Department of Mathematics

Overview

The concept of a limit is an idea that all other calculus concepts are based on. Limits involving infinity are closely related to asymptotes. While asymptotes for functions are sometimes easy to identify from a graph, the actual definitions of asymptotes is in terms of limits. There are many different types of asymptotes, and the two simplest are:

| Asymptote | Equation | Definition |
|------------|----------|--|
| Horizontal | $y = L$ | $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$ |
| Vertical | $x = a$ | $\lim_{x \rightarrow a^+} f(x) = \infty$ (or $-\infty$) or $\lim_{x \rightarrow a^-} f(x) = \infty$ (or $-\infty$) |

This lab is designed to provide experience finding asymptotes. Some Simple limits can easily be evaluated by inspection. You will learn several ways to use Maple to help evaluate more difficult limits.

Maple Essentials

Important Maple commands introduced in this lab:

| Command | Description | Example |
|-------------------------------------|--|---|
| <code>limit(f(x),x=a);</code> | evaluate $\lim_{x \rightarrow a} f(x)$ | <code>limit(f(x),x=2);limit(x^2,x=infinity);</code> |
| <code>limit(f(x),x=a,right);</code> | evaluate $\lim_{x \rightarrow a^+} f(x)$ | <code>limit((f(x)+1)/(x^2-1),x=1,right);</code> |
| <code>limit(f(x),x=a,left);</code> | evaluate $\lim_{x \rightarrow a^-} f(x)$ | <code>limit(sqrt(x^2+1)/(x+1),x=-1,left);</code> |
| <code>solve</code> | solve equations | <code>solve(x^2=8,x);solve(denom(f(x))=0,x);</code> |

Related Text

§2.2, §2.3, and §2.6 of the Calculus Text and §4.1 of the Maple Text

Activities

- Use Maple to find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for the following functions.
 - $f(x) = x^2$
 - $f(x) = \sqrt{x}$
 - $f(x) = 1/x$
 - $f(x) = \sin x$

Note: You will soon learn that this limit defines the derivative of $f(x)$.

- Identify all horizontal and vertical asymptotes for as many of the functions on the back of this page as possible.

General Directions

1. Define your function, say $f(x)$, using `f:=x->` and verify that it is entered correctly.
2. Look at the function $f(x)$ and determine which values make the denominator zero. These values will be the a 's that we need to check as possible vertical asymptotes. If you need help, you can use the `solve` command as follows:
`> solve(denom(f(x))=0,x);`
3. To find the vertical asymptotes, you need to evaluate the one-sided limits at $x = a$. Enter the following lines of code:
 (a) `limit(f(x), x=a, left);`
 (b) `limit(f(x), x=a, right);`
 If either of these returns the value ∞ or $-\infty$ then $x = a$ is the equation of a vertical asymptote of $f(x)$.
4. To find the horizontal asymptotes of $f(x)$, you need to find the limits at ∞ and $-\infty$. Enter the following lines of code:
 (a) `limit(f(x), x=infinity);`
 (b) `limit(f(x), x=-infinity);`
 If either of these returns a value $L \neq \pm\infty$ then $y = L$ is the equation of a horizontal asymptote of $f(x)$.
5. Plot the function on a standard window and make sure your answers are consistent with the graph. Use the following line of code:
`> plot(f(x), x=-10..10, y=-10..10, discontin=true);`

Functions

1. $f(x) = \frac{5+2x}{x+1}$
2. $f(x) = \frac{3x+2}{(x+2)^2}$
3. $f(x) = \frac{(x^2-1)\sqrt{4x^2+1}}{x^3-2x^2-x+2}$
4. $f(x) = \frac{\sin x}{x}$
5. $f(x) = \frac{x^3+3x^2-12x+4}{x^2}$
6. $f(x) = \frac{x^3+3x^2-12x+4}{x^3-x^2-4x+4}$
7. $f(x) = \frac{\sqrt{x^2+1}+2x}{2x-3}$
8. $f(x) = \frac{\sqrt{9x^2+4}-2}{x+1}$

Assignment

For your assignment this week, complete the following. Turn in a sheet of paper with your answers and attach your Maple worksheet as supporting work.

1. Answer each of the following questions. Explain all of your answers and use examples from today's lab where appropriate.
 - (a) Does every undefined value of $f(x)$ lead to a vertical asymptote?
 - (b) Can the graph of a function cross the graph of its horizontal asymptotes?
 - (c) Can the graph of a function cross the graph of its vertical asymptotes?
 - (d) How many horizontal asymptotes can the graph of a function have?
2. Identify all horizontal and vertical asymptotes for the following functions.
 - (a) $f(x) = \frac{6x^2-10x-4}{9x^3+27x^2-x-3}$
 - (b) $h(x) = \frac{\sqrt{16x^2+3}-8x}{4x+1}$