Differentiation and Tangent Lines

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Overview

This week's lab will provide practice finding locally linear approximations to functions. That is, we will be finding the equation of the tangent line to a curve.

Maple Essentials

- The *Tangents* tutor is started from the Maple 10 user interface under the Tools menu:

  Tools → Tutors → Calculus - Single Variable → Tangents...

- The *TangentLine* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-F06/labs/` → TangentLine

- The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

Command	Description
`:=`	assignment
`x ->`	function definition
`plot`	plot one or more expressions

Preparation

Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line and m is the slope of the line. Next, solve the equation for y and we get:

$$y = m(x - x_1) + y_1.$$

Now, we use the substitution $y_1 = f(x_1)$ and this becomes:

$$y = m(x - x_1) + f(x_1).$$

Finally, we know that the derivative evaluated at x_1 is the same as the slope of the tangent line at x_1. Thus we get the following formula for the equation of the tangent line at x_1:

$$y = f'(x_1)(x - x_1) + f(x_1).$$

Assignment

This week's mastery quiz asks you to find and graph the tangent line for a given function. The Activities in this lab will help you answer the Mastery Quiz questions. The *TangentLine* maplet provides additional practice finding tangent lines.

Activities

We will find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$ for several different functions. We will then graph the function and its tangent line on the same axes.

Example Problem

We will solve the following problem together in two different ways:

- Find an equation for the line that is tangent to the graph of the differentiable function $f(x) = x^3 - 2x + 1$ at $x_1 = 2$. Then graph the curve and this tangent line on the same axes.

The first way:

1. Launch the *Tangents* tutor.
2. Enter the function as x^3-2*x+1 and x=2, and change the number of iterations to 5.
3. Click **Display**. The tutor will display the function and a series of secant lines, including the tangent line. The equation of the tangent line is displayed on the right.
4. Press the **Animate** button. The tutor will show the progression through the secant lines as Δx gets smaller.
5. The tutor will return the last graph when you click **Close**.
6. If you want to graph the function and the tangent line, assign both in a Maple worksheet and write a plot command.

The second way:

1. Define and assign the function to f.
   `> f := x -> x^3-2*x+1;`
2. Right-click and choose *Differentiate*. Then use a label to assign this new function to *df*.
   `> df := label;`
3. Find $f'(2)$ and assign that value to m.
   `> m := df(2);`
4. Find the tangent line $y = f'(2)(x - 2) + f(2)$ and assign it to L.
   `> L := m*( x-2 ) + f(2);`
5. Plot the function and the tangent line using different linestyles.
   `> plot( [f(x), L], x=-2..3, linestyle=[SOLID, DOT]);`

Functions

Find the equation of the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$. Graph the function and its tangent line on the same axes.

1. $f(x) = \sqrt{x}, x_1 = \frac{1}{4}$
2. $f(x) = \frac{5}{x} + 1, x_1 = -2$
3. $f(x) = x^2, x_1 = 1$
4. $f(x) = 2^x, x_1 = 3$
5. $f(x) = \cos(x), x = \frac{\pi}{4}$

Additional Notes

- The `diff` command can also be used to evaluate higher order derivatives. For example, if we wanted to find the second derivative of `F`, we would use the command `diff( F, x, x )`. This notation, however, becomes somewhat tedious for higher order derivatives. The command `x$n` repeats `x` successively `n` times and thus shortens our commands. For example, for the fourth derivative of `F`, we could write `diff( F, x$4 )` instead of `diff(F, x, x, x, x )`.