MATH 554 - ANALYSIS I
TEST 3 – DUE TUES, NOV. 25

1	(10 *pts*)
2	(15 *pts*)
3	(10 *pts*)
4	(20 *pts*)
5	(15 *pts*)
6	(10 *pts*)
7	(15 *pts*)
8	(10 *pts*)
	(105*pts*)

Name: ____________________________

Directions: Answers should be provided in complete sentences with justifications.

1. Suppose $f : A \to B$ where $(A, d_A), (B, d_B)$ are metric spaces.
   a) Give the definition for a function f to be continuous at a point $x_0 \in A$.

   b) If x_0 is an isolated point of A, using the definition, prove that every function $f : A \to B$ is continuous at x_0.

2. Suppose $f : A \to B$ is a continuous function at a point x_0, then prove that whenever a sequence $\{x_n\}_{n=1}^{\infty}$ is a sequence which converges to x_0, then the corresponding $y_n := f(x_n)$ form a sequence in B which converges to the point $y_0 := f(x_0)$.

3. Prove that the composition of two continuous functions is continuous.
(*Note:* You may prove this using any of the four characterizations of continuity that we had in our theorem, i.e. sequences, inverse images of open sets, ...)

4. Consider a metric space A and a subset $C \subseteq A$.
a) Define a *disconnection* for a subset C.

b) Define what it means for a set C to be *connected*.

c) Give an example of a subset of the real numbers which is connected and one which is not.

d) What precisely are the connected subsets of the real numbers?

5. Let $R(x)$ be any rational function on the real numbers (i.e. R is a quotient of polynomials).
a) What is the domain of R and at which points x is R continuous?

b) What is the domain of $f(x) := \sqrt{R(x)}$ and where is it continuous?

6. State and prove the Intermediate Value Theorem for continuous functions.

7. a) Define what it means for a set $K \subseteq A$ to be a compact set.

   b) Give an example of a subset of the real numbers which is compact and one which is not.

8. State the Heine-Borel Theorem.