Math 554
Handout #4 - part B

Example. Each of the following is an example of a closed set:

a.) Each closed interval $[c,d]$ is a closed subset of $\mathbb{R}$.

b.) The set $(-\infty, d\,] := \{x \in \mathbb{R} |\; x \leq d\}$ is a closed subset of $\mathbb{R}$.

c.) Each singleton set $\{x_0\}$ is a closed subset of $\mathbb{R}$.

d.) The *Cantor set* is a closed subset of $\mathbb{R}$.

To construct this set, start with the closed interval $[0,1]$ and recursivley remove the open middle-third of each of the remaining closed intervals ...

At the n-th stage, we have a set F_n consisting of 2^n closed intervals each of length $(\frac{1}{3})^n$:

Stage 0:	$[0,1]$			
Stage 1:	$[0,\frac{1}{3}]$		$[\frac{2}{3},1]$	
Stage 2:	$[0,\frac{1}{9}]$	$[\frac{2}{9},\frac{3}{9}]$	$[\frac{6}{9},\frac{7}{9}]$	$[\frac{8}{9},1]$

$\vdots$

This finite union of closed intervals is closed. The *Cantor set* is the intersection of this (decreasing, or nested) sequence of sets and so is also closed. Later, we will see that it has many other interesting properties.

We also showed in class that this set consists of real numbers whose ternary series expansions have coefficients which are drawn from $\{0,2\}$, which allowed us to observe that the Cantor set is uncountable. Finally we showed that the 'length' of the corresponding set F_n at Stage n is $(2/3)^n$, so the Cantor set has 'length' equal $\lim_{n\to\infty}(2/3)^n = 0$.