MATH 554/703I - ANALYSIS I
HOMEWORK #4
DUE THURSDAY, OCT 23 2008

1. Show each Cauchy sequence is bounded.

2. Suppose $\{a_n | n \in \mathbb{N}\}$ is a Cauchy sequence in $\mathbb{R}$, then show that

$$\alpha_N := \operatorname{lub}_{n \geq N} a_n$$

   is a monotone decreasing sequence which converges.

3. If $\{p_n | n \in \mathbb{N}\}$ is a Cauchy sequence and p_0 is a limit point of the sequence considered as a set, then show that $\lim\limits_{n \to \infty} p_n = p_0$.

4. Carefully determine whether the sequences $\{a_n\}$ converge and, if so, determine their limits using the standard properties involving sequences: i.e. sums, products, and quotients.

   (a) $a_n := 15/n$

   (b) $a_n := \dfrac{(-1)^n + 1}{n^2}$

   (c) $a_n := \dfrac{n^2 - 3n}{5 - 3n^3}$

5. Prove that if $\lim\limits_{n \to \infty} a_n = 0$ and $\{b_n\}$ is bounded, then $\{a_n b_n\}$ converges.

6. Prove the *reverse triangle inequality* for absolute value:

$$|a| - |b| \leq |a - b|$$

7. Using the previous problem, show that if $\lim_{n \to \infty} b_n = b$ and $b \neq 0$, then there exists $N \in \mathbb{N}$ so that for every $n \geq N$, the lower estimate

$$|b_n| > |b|/2$$

   is satisfied.