MATH 172 Spring, 2004 Exam #1 Name:____________________

There are 100 points. For full credit you must show your work. You may use a calculator, but this does not exempt you from explaining your answers by giving results of computations or sketches of graphs, etc.

1. (25 points) a. Formulate a model (dependent variable u, independent variable n) in which 60% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 60 mg/day. The initial dose is 70 mg.

   b. What is the steady state amount of the drug in the bloodstream?

   c. Compute u_1, u_2, u_3, u_5, u_{10}.

   d. Describe the long term behavior of u_n if the initial dose is instead $u_0 = 125$ mg. Does u_n increase, decrease, oscillate, tend towards or away from the equilibrium?

   e. Is the equilibrium value you found in part (b) stable or unstable? Explain verbally and / or graphically.

2. (20 points) A population $B(t)$ of bacteria is growing **continuously** over time so that the **per capita** rate of increase is 0.071 /day. (We don't count bacteria, we weigh them.)

   a. Write the model equation that describes this situation.

   b. If $B(0) = 5$ g (or 5000 mg), write the solution equation, for this model equation. This initial population of 5 g grows to what size in 31 days?

   c. If instead of bacteria, B_t represented female bison population, which reproduce only once a year, with a discrete growth rate of 0.071 /year, determine B_{31} if $B_0 = 5$.

3. (20 points) A chemical reaction involves constituents A, B, and C, in which C acts as an enzyme catalyzing the conversion of A into B. Each minute 45% of A is converted into B, while 10% of B converts naturally back into A. Meanwhile 5% of the enzyme C is used up. To keep the system running 25 units of A and 10 units of C are added each minute. Write model equations for a_n , the amount of A, b_n , the amount of B, and c_n , the amount of C, at time n minutes.

4. (10 points) Give the "model equation" ("dynamic approach") and a "solution equation" ("explicit approach") for the height h_n of a stack of n chairs, where each chair is 3 feet high, and when you stack a new chair onto the pile, only 8 inches sticks out above the previous chair. Note that the pattern doesn't really begin until you actually have one chair, so h_0 is not defined and $h_1 = 3$. Be clear and consistent with your units!

5. (15 points) Here is a table of values for a 2-variable dynamical system

n	0	1	2	3	4	5	6	7	8
u_n	3	1	5	6	3	1	5	6	3
v_n	4	8	9	3	4	8	9	3	4

Plot u_n and v_n against one another on one graph, and label the points with the values of n from 0 to 9. Plot u_n and v_n on a single graph against n from 0 to 8. What conclusion do you draw about this dynamical system, at least with the initial condition $u_0 = 3$ and $v_0 = 4$?

6. (10 points) Compute the equilibrium point (E, F) of the dynamical system

$$\begin{aligned} u_n &= 3u_{n-1} - 2v_{n-1} - 4 \\ v_n &= 5u_{n-1} - 3v_{n-1} - 28 \end{aligned}$$

7. (Bonus 10 points) a. Describe the long term behavior of the solution to the dynamical system $w_n = -0.2w_{n-1} + 24$, $w_0 = 40$; that is, what happens to w_n as $n \to \infty$? Does w_n increase, decrease, oscillate, tend towards or away from the equilibrium?

b. What is the equilibrium value and is it stable or not? Explain.

c. Express w_{n+1} in terms of w_n.