MATH 172 Fall, 2011 Quiz #5 Name:

Recall that (1) an affine continuous model $\frac{dQ}{dt} = aQ + b$ has an explicit solution $Q(t) = Ce^{at} + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition, and (2) an affine discrete model $Q_{n+1} = aQ_n + b$ has an explicit solution $Q_n = Ca^n + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition.

- 1. A population of whales W = W(t) growing continuously at a **per capita rate** of 2% yr⁻¹, but 6 migrate away over the course of each year.
 - a. Write an affine model equation for this situation, and solve it, assuming that the initial whale population is 250.

b. What exactly happens to the whale population in the long term, and how do you know? If the population is growing, compute the doubling time; if the population is shrinking, compute the extinction time.

- 2. The growth rate of a population N = N(t) is governed by the model equation $\frac{dN}{dt} = 0.04N \left(1 - \frac{N}{150}\right) \left(\frac{N}{30} - 1\right).$ a. Determine the equilibrium values.

 - b. If N(0) = 35, sketch the long term behavior of N(t), and explain why the graph has the shape that it does. Then do the same if N(0) = 25.

c. Say whether each equilibrium value is stable or unstable.