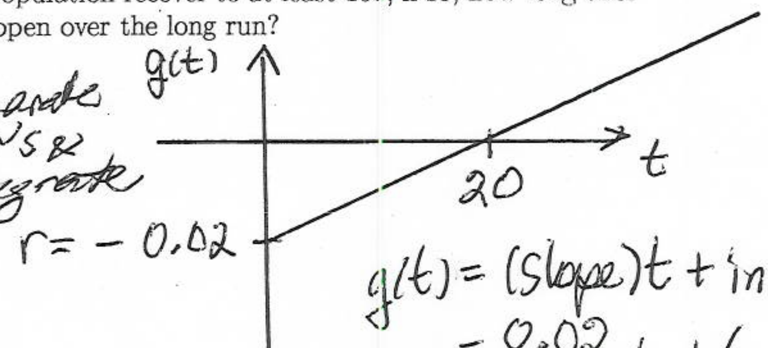
MATH 172 Fall, 2009 Quiz #4 Name: MR

This is a take home quiz, due Tuesday, 29 September in class. You may work together to get ideas, but you must write up your solution by yourself.

1. A population P_t of salamanders is down to 100, when the Nature Conservancy takes over the habitat of land and ponds and begins environmental remediation work. Polluted storm water and industrial drainage is diverted, invasive fish are removed, indigenous plants are replanted, and so on. The per capita growth rate $g(t)$ of the salamander population, now at $r = -0.02$, is assumed to increase linearly in such a way that at $t = 20$ we have $g(20) = 0$, and thereafter $g(t)$ becomes positive. Find the formula for $g(t)$ and plot it. Our model takes the form $P'(t) = \frac{dP}{dt} = g(t)P$. Use separation of variables to get a formula for $P(t)$. How well does the model predict that the plan will do? You may want to produce a graph of $P(t)$ (put your calculator back in Func Mode, choose an appropriate range of values for x and plot $Y = P(x)$), or you may use your formula and just plug in different values for time. How low does the population decline, and when does this happen? Does the population recover to at least 100; if so, how long does this take? What seems to happen over the long run?

$\frac{dP}{dt} = g(t)P$ separate var's & integrate



$\int \frac{1}{P}\,dP = \int g(t)\,dt$

$g(t) = (\text{slope})t + \text{intercept}$
$= \frac{0.02}{20}t + (-0.02)$
$= 0.001t - 0.02$

$\ln P = \int (0.001t - 0.02)\,dt$

$= 0.001(\frac{1}{2}t^2) - 0.02t + C$ Set $A = e^C$.

$P(t) = e^{0.0005t^2 - 0.02t} \cdot e^C = Ae^{0.0005t^2 - 0.02t}$

Plug in $t=0$: $P(0) = 100 = Ae^0$ so $A = 100$.

Then $P(t) = 100e^{0.0005t^2 - 0.02t}$

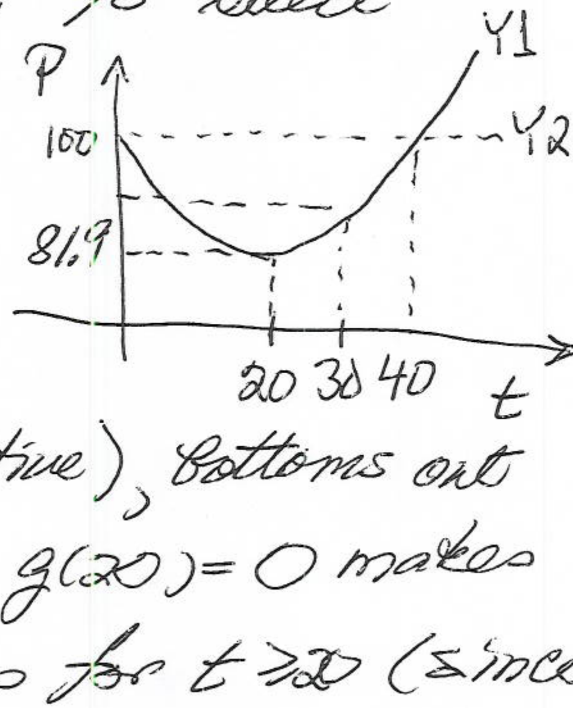
$P(5) = 91.6$, $P(10) = 86.1$, $P(15) = 82.9$,
$P(20) = 81.9$, $P(25) = 82.9$, $P(30) = 86.1$,
$P(35) = 91.6$, $P(40) = 100$, $P(45) = 111.9$

(over)

If you switch your MODE back to Func, and put in

$$y = 100e^{0.0005x^2 - 0.02x}$$

you can graph this, with say a window $x = -5$ to $x = 45$ and $y = 60$ to $y = 120$, you can see in detail what is happening continuously. You can use TRACE to select certain x values.



You'll notice that $P(t)$ decreases for $0 \le t \le 20$ (since $\frac{dP}{dt} = g(t)P$ is negative), bottoms out at $t = 20$ (because $g(20) = 0$ makes $\frac{dP}{dt} = 0$), and rises for $t \ge 20$ (since $\frac{dP}{dt} = g(t)P$ is positive). The pop. has not recovered by $t = 30$, but is on its way up, recovers at $t = 40$, and then grows very quickly.