MATH 172 Fall, 2009 Exam #1 Name:____________________

There are 110 points, which will be scaled down to a percentage (out of 100). **For full credit you must show your work**. All answers should be **correctly rounded to 3 decimal places**, or fewer if that is what is provided in the data.

1. (10 points) If $Q(n) = 5n - n^2$, compute ΔQ .

2. (10 points) Papua New Guinea currently has a population of 56.7 hundred thousand people. The **net** growth rate is 1.253 hundred thousand people per year. Compute the intrinsic or **per capita** growth rate r . Write a continuous model equation for this process. Give the explicit solution and use your result to predict the population a decade from now.

3. (15 points) A population $B(t)$ of bacteria is growing **continuously** over time so that the **per capita** rate of increase is 6% /day.
   a. Write the continuous model equation that describes this situation.

   b. If $B(0) = 5\,\text{g}$, write the solution equation for this model equation. (We don't count bacteria, we weigh them.) This initial population of 5 g grows to what size in 3 weeks?

4. (20 points) A population R of rats kept in a lab depends on t. The experimenter is testing the effect of a poison in the food. Observation takes place at regular weekly intervals. At the start of the experiment ($t = 0$), the population is 400. Week by week, the population shrinks by 9%.

   a. Write the difference equation for this discrete model.

   b. Write the updating equation for this model.

   c. Write the explicit solution for this model, and use this to compute the population after 6 months (26 weeks).

   d. At what time is the population 1/2 of the original?

5. (10 points) Due to a parasite infecction that increases mortality and decreases reproduction, a host population $H(t)$ is decreasing at a continuous rate of 4% yr^{-1}. Uninfected organisms move into the habitat at a rate 100/year, become infected, and reproduce and die as do the residents already there. Write the continuous model equation to describe this process, and compute the equilibrium value, if there is one.

6. (25 points) A drug is given in pill form (160 mg) once a day. We know that 80% of drug in the bloodstream is used up over the day and through the night, but the remainder is reinforced by a maintenance dose of one pill early in the morning. Let A_t denote the amount of the drug in mg after t days, after the drug level has fallen and the morning pill has been taken The initial dose, given late in the day, is one half pill (80 mg); *i.e.*, $A_0 = 80$.

   a. Formulate a discrete model to describe this process over time. Compute A_1 and A_2.

   b. What is the steady state (equilibrium) amount of the drug in the bloodstream?

   c. In the calculator version of this model A_t is replaced by $u(n)$. It has $n\text{Min} =$ ______ , $u(n\text{Min}) =$ ______ , and $u(n) =$ (______)u(______) + (______).

   d. Compute $A_3 = u_3$, $A_5 = u_5$, $A_{10} = u_{10}$ by hand or by calculator or both. What is the long term trend?

   d. (Bonus) Is the equilibrium value you found in part (b) stable or unstable? Explain verbally and / or graphically.

7. (20 points) An algal population $A(t)$, measured in tons, grows in some lakes and rivers that accumulate nutrients from agricultural runoff. It is governed by the continuous model $\frac{dA}{dt} = 0.0003A(200 - A) = 0.06A(1 - \frac{A}{200})$ tons/year. The initial population is $A(0) = 50$ tons. Since we are measuring in tons, it is reasonable that you may get decimal answers; round off to **three** decimal places.

a. Apply stepwise estimation using Euler's method to estimate the population after 12 years. Remember that $\Delta A \approx \frac{dA}{dt}(\Delta t)$. Use 3 steps, so each time step is $\Delta t =$ ______ to approximate $A(12)$, and show your work! Here is the outline of a table to help you get started. Final answer: $A(12) \approx$ ________________________________ .

n	t	$A(t)$	$A'(t)$	$\Delta A \approx A'(t)\Delta t$	$A(t + \Delta t)$
0	0	50	2.25	9	59
1		59			

b. If we want to approximate $A(12)$ again, but use $\Delta t = 1/4 = 0.25$ year (that is, 3 months), how many steps are required to get to 12 years? ______ .