MATH 172 **Fall, 2007** **Exam #1** **Name:** M^2

There are 100 points. **For full credit you must show your work.** All answers should be **correctly rounded to 3 decimal places**, or fewer if that is what is provided in the data.

1. (12 points) If $U(n) = n^2 - 4n$, compute ΔU.

$$\Delta U = U(n+1) - U(n)$$
$$= [(n+1)^2 - 4(n+1)] - [n^2 - 4n]$$
$$= n^2 + 2n + 1 - 4n - 4 - n^2 + 4n$$
$$= 2n - 3$$

adding a constant in each step yields linear growth

2. (12 points) A quantity $Q(n)$ is governed by the discrete dynamic model equation $\Delta Q = 0.4$. If we observe that $Q(2) = 3$, find an explicit formula for $Q(n)$ in terms of n (in other words the solution or static model equation).

$$Q(n+1) - Q(n) = 0.4$$
$$Q(n+1) = Q(n) + 0.4$$

n	Q
0	2.2
1	2.6
2	3
3	3.4
4	3.8
⋮	
n	$3 + (n-2)(0.4)$

$3.4 = 3 + (.4)(1)$

$3.8 = 3 + (2)(.4)$

By backing up in the table we see $Q(0) = 2.2 = 3 - (2)(0.4)$

So $\boxed{Q(n) = 2.2 + 0.4n}$

Or by reading forwards we see

$$\boxed{Q(n) = 3 + (n-2)(0.4) = 0.4n + 2.2}$$

3. (20 points) A population B of insects kept in a lab depends on t. Observation takes place on a regular one day interval. At the start of the experiment ($t = 0$), the population is 500, and the change in B is observed to be governed by the discrete model equation $\Delta B = -0.05B(t)$. Write the updating (recurrence) model equation for B and determine an explicit formula (static solution) for $B(t)$ in terms of t. How many insects can we predict will be observed on the 120^{th} day? What happens to this population in the long run?

$$\Delta B = B(t+1) - B(t) = -0.05B(t)$$

$$B(t+1) = B(t) - 0.05B(t) = (1-0.05)B(t)$$

Having a constant multiplier in each step gives exponential growth. $= (0.95)B(t)$

$$\boxed{B(t) = B(0)(0.95)^t = (500)(0.95)^t}$$

$$B(120) = (500)(0.95)^{120} \approx 1 \text{ whole insect}$$

Since $(0.95)^t \to 0$ as t gets larger and larger, this population will die out.

4. (16 points) A chemical reaction in a beaker of water involves water soluble chemicals X and Y that interact continuously. In the presence of Y, the chemical X converts into Y at a rate proportional to the amount of Y. The amount of X in the solution is continuously increased at a constant rate m by the experimenter. No Y is converted back to X, but it is converted into water at a rate proportional to the amount of X. Write a continuous dynamic model system that describes this process of the reaction of the two chemicals.

indep. variable is t

$$\xrightarrow{m} \frac{dX}{dt} \xrightarrow{aY} \frac{dY}{dt} \xrightarrow{bX} H_2O$$

rates of change

$$\begin{cases} \dfrac{dX}{dt} = m - aY \\[2ex] \dfrac{dY}{dt} = aY - bX \end{cases}$$

a, b, m constants (parameters)

5. (20 points) The **net** rate of change of a continuously growing fruit fly population is directly proportional to the population $F(t)$ itself. At the beginning of the experiment when there are 1000 flies, we observe that this rate is 14 flies/week.

a. Determine the value of the constant of proportionality r. In this case the **per capita** rate of change (with units) has the value $0.014/$ week. Give the dynamic model equation using the variables F, t, and explicit constants.

$$\frac{dF}{dt} = rF \qquad (14 \text{ flies/week}) / 1000 \text{ flies}$$

$$r = \frac{dF}{dt} \Big/ F \qquad = 0.014 / \text{week}$$

$$\boxed{\frac{dF}{dt} = 0.014 F(t)}$$

b. Your lab partner claims that the static model equation for this population is $F(t) = 1000e^{0.28t}$. If so, then $F(0) =$ 1000 and the dynamic model equation is the one you gave in part a. Is your partner's claim fully correct, partially correct, or entirely wrong, and how can you demonstrate this?

$F(0) = 1000e^{0} = 1000$ is correct.

$$\frac{dF}{dt} = 1000(0.28)e^{0.28t} \text{ by calculus}$$

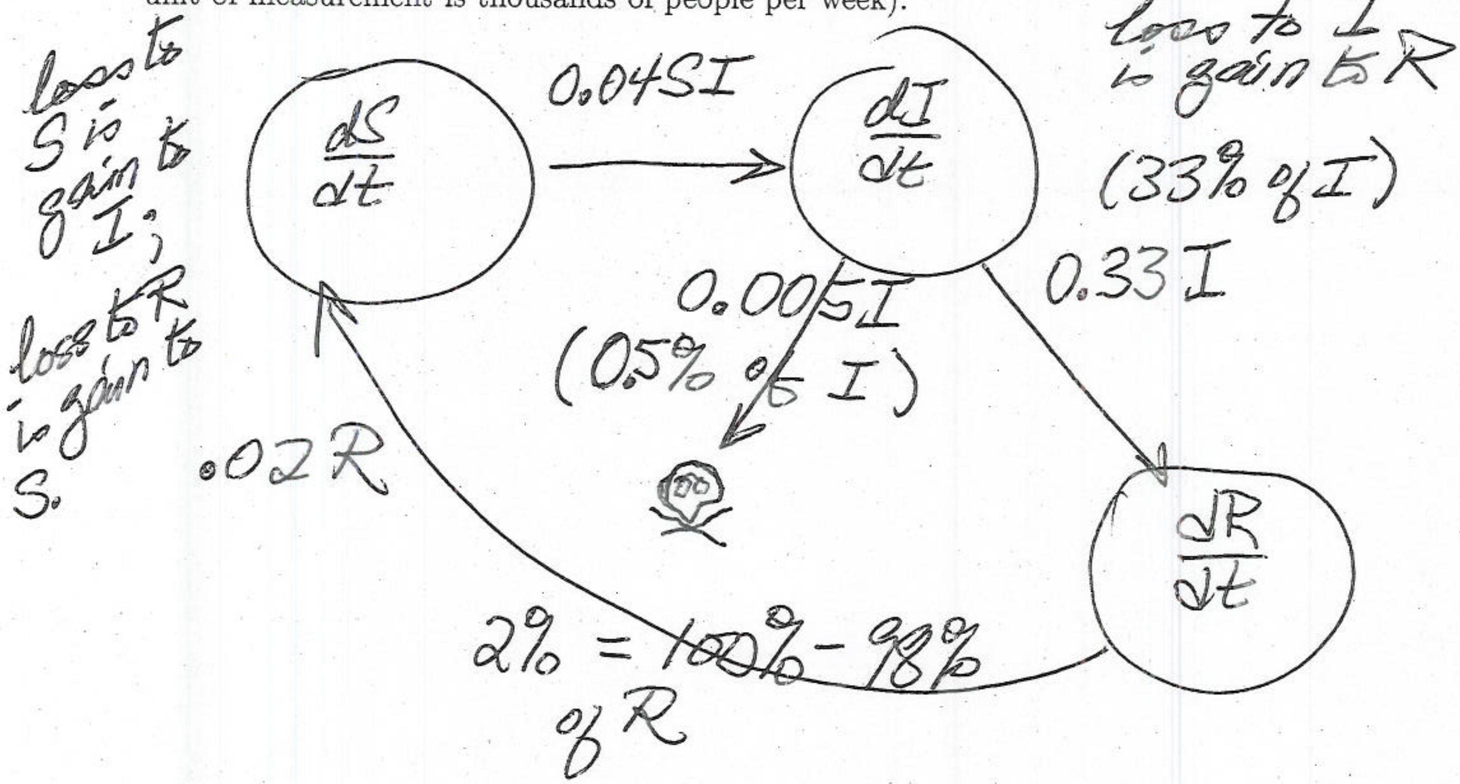
$$= (0.28)(1000e^{0.28t})$$

$$= (2)(0.14)F(t) = (2)(10)(0.014)F(t)$$

$$= (20)(0.014F(t))$$

$\neq 0.014 F(t)$ which is what the rate of change should be from (a). Partner is only partially correct.

6. (20 points) A not very infectious illness, but one that is occasionally fatal, is spreading through a susceptible population $S(t)$. Each day there is a mass action interaction with transmission coefficient 0.04 of the susceptible population $S(t)$ with the ill and infectious population $I(t)$ in which susceptible individuals become ill and infectious. At the same time 33% of the infectious population recovers; the recovered population is denoted $R(t)$. The time span is short enough that no one dies of other causes in the period under observation, but 0.5% of the infected population dies. Most of the recovered population, in fact 98%, is immune to reinfection, but the rest do become susceptible again. Write a system of continuous dynamic model equations for for this process (the unit of measurement is thousands of people per week).



$$\frac{dS}{dt} = -0.04SI + 0.02R$$

$$\frac{dI}{dt} = +0.04SI \underbrace{-0.33I - 0.005I}_{-0.335I}$$

$$\frac{dR}{dt} = 0.33I - 0.02R$$