MATH 550 (Section 501)
Prof. Meade

University of South Carolina
Spring 2009

Exam 1
February 19, 2009

Name: ____________________
Section 501

Instructions:

1. There are a total of 7 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	10	
2	10	
3	20	
4	20	
5	12	
6	12	
7	16	
Total	100	

Good Luck!

1. (10 points) A bug finds itself in a toxic environment. The toxicity level is given by $T(x, y) = 2x^2 - 4y^2$. The bug is at $(-1, 2)$. In what direction should the bug move to lower the toxicity the fastest?

2. (10 points) Write an iterated triple integral that represents the volume of the solid bounded by $x/a + y/b + z/c = 1$ and the coordinate planes.

   NOTE: Do not attempt to evaluate this integral.

3. (20 points) Let $\mathbf{F}(x, y) = f(x^2 + y^2)(-y\mathbf{i} + x\mathbf{j})$.

   (a) Calculate $\nabla \cdot \mathbf{F}$.

   (b) Calculate $\mathbf{curl}\,\mathbf{F}$.

4. (20 points) Let B be the region in the first quadrant bounded by the curves $xy = 1$, $xy = 3$, $x^2 - y^2 = 1$, and $x^2 - y^2 = 4$.

   (a) Graph B.

   (b) Graph $B^* = T(B)$ for the change of variables given by $u = x^2 - y^2$ and $v = xy$.

   (c) Evaluate $\iint_B (x^2 + y^2)\ dx\ dy$.

5. (12 points) Find the volume of the solid bounded below by $x^2 + y^2 = z$ and above by $x^2 + y^2 + z^2 = 2$.

6. (12 points) Suppose the density of a sphere of radius R is given by $\delta = (1 + d^3)^{-1}$ where d is the distance to the center of the sphere. Find the total mass of the sphere.

7. (16 points) Consider the double integral $\iint_D xe^{-y^3}\,dx\,dy$ where $D = \{(x, y)|0 \le x \le y < \infty\}$.

   (a) Why is this an improper integral?

   (b) Evaluate the integral.

   NOTE: Clearly identify the proper integral that you evaluate and the limits used to complete the evaluation of the improper integral.