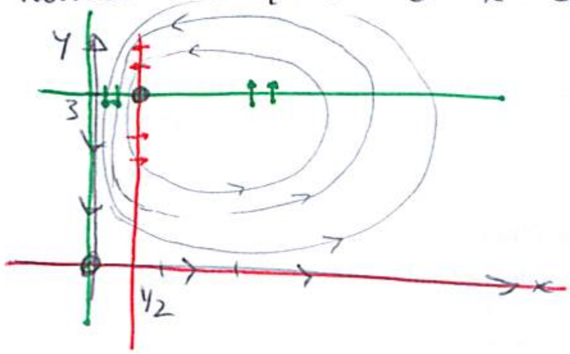
HW Solutions - §9.5

#1. $x' = x(1.5 - 0.5y)$
$y' = y(-0.5 + x)$

(a) — x nullcline : $x = 0$ or $y = 3$
— y nullcline : $y = 0$ or $x = 0.5$



(b) critical points are $(0,0)$ and $(\frac{1}{2}, 3)$.

(c) $J = \begin{pmatrix} 1.5 - 0.5y & -0.5x \\ y & -0.5 + x \end{pmatrix}$

so $J(0,0) = \begin{pmatrix} 1.5 & 0 \\ 0 & -0.5 \end{pmatrix}$ and $(0,0)$ is an unstable saddle point.

$J(\frac{1}{2}, 3) = \begin{pmatrix} 0 & -1/4 \\ 3 & 0 \end{pmatrix}$ and $(\frac{1}{2}, 3)$ is a stable center. (in the linear system)
$(p = 0,\ q = \frac{3}{4},\ \Delta = -\frac{9}{16})$

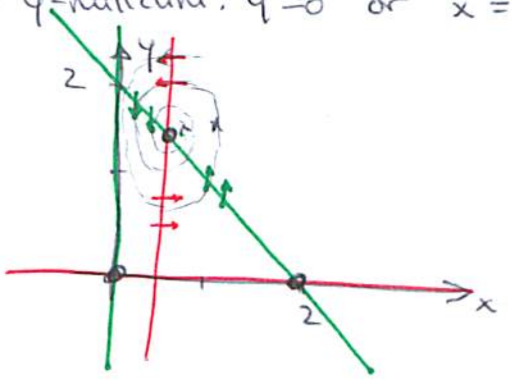
(f) Solutions in the first quadrant move along closed curves that move counter-clockwise around $(\frac{1}{2}, 3)$.

These solution curves are horizontal $(y' = 0)$, pointing right when $x = \frac{1}{2}, y < 3$ and pointing left when $x = \frac{1}{2}, y > 3$.
They are vertical $(x' = 0)$, pointing straight up when $y = 3, x > 1/2$ and pointing straight down when $y = 3, x < \frac{1}{2}$.

#3. $x' = x(1 - \frac{x}{2} - \frac{y}{2})$
$y' = y(-\frac{1}{4} + \frac{x}{2})$

Note: This problem is the predator-prey model with logistic growth in the prey population (with carrying capacity 2). See how this changes the nature of the solutions?

(a) — x-nullcline: $x=0$ or $x+y=2$
— y-nullcline: $y=0$ or $x=\frac{1}{2}$



(b) critical points: $(0,0)$, $(2,0)$, $(\frac{1}{2}, \frac{3}{2})$

(c) $J = \begin{pmatrix} 1 - x - \frac{y}{2} & -\frac{x}{2} \\ \frac{y}{2} & -\frac{1}{4} + \frac{x}{2} \end{pmatrix}$

so
$J(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}$ and $(0,0)$ is an unstable saddle point.

$J(2,0) = \begin{pmatrix} -1 & -1 \\ 0 & \frac{3}{4} \end{pmatrix}$ and $(2,0)$ is an unstable saddle point.

$J(\frac{1}{2}, \frac{3}{2}) = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{3}{4} & 0 \end{pmatrix}$ with $p = -\frac{1}{4}$, $q = \frac{3}{16}$, $\Delta = -\frac{11}{16}$ we see that $(\frac{1}{2}, \frac{3}{2})$ is an asymptotically stable spiral.

(f) solutions that start in the 1st quadrant spiral, counter-clockwise, towards the origin as $t \to \infty$.
Note that solution curves are horizontal, pointing to the right when $x = \frac{1}{2}$, $y < \frac{3}{2}$ and pointing to the left when $x = \frac{1}{2}$, $y > \frac{3}{2}$. Also, solutions are vertical, pointing down when $x+y=2$, $x < \frac{1}{2}$ and pointing up when $x+y=2$, $x > \frac{1}{2}$.