

HW Solution - §7.2

$$\#23. \vec{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$$

$$\text{To verify that } \vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t e^t = \begin{pmatrix} 2t+1 \\ 2t \end{pmatrix} e^t$$

satisfies the DE we compute

$$1) \text{ the LHS: } \vec{x}' = \begin{pmatrix} 2 \\ 2 \end{pmatrix} e^t + \begin{pmatrix} 2t+1 \\ 2t \end{pmatrix} e^t = \begin{pmatrix} 2t+3 \\ 2t+2 \end{pmatrix} e^t$$

$$\begin{aligned} 2) \text{ the RHS: } \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t &= \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 2t+1 \\ 2t \end{pmatrix} e^t + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t \\ &= \begin{pmatrix} 2t+2 \\ 2t+3 \end{pmatrix} e^t + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t \\ &= \begin{pmatrix} 2t+3 \\ 2t+2 \end{pmatrix} e^t \end{aligned}$$

$$\#26. \vec{\Psi}' = \begin{pmatrix} 1 & 1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \vec{\Psi}$$

$$\text{To verify that } \vec{\Psi} = \begin{pmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{pmatrix} \text{ satisfies}$$

the DE we compute

$$1) \text{ the LHS: } \vec{\Psi}' = \begin{pmatrix} e^t & -2e^{-2t} & 3e^{3t} \\ -4e^t & 2e^{-2t} & 6e^{3t} \\ -e^t & 2e^{-2t} & 3e^{3t} \end{pmatrix}$$

$$2) \text{ the RHS: } \begin{pmatrix} 1 & 1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{pmatrix} = \begin{pmatrix} e^t & -2e^{-2t} & 3e^{3t} \\ -4e^t & 2e^{-2t} & 6e^{3t} \\ -e^t & 2e^{-2t} & 3e^{3t} \end{pmatrix}$$