HW Solutions - §3.1

#7. $y'' + 9y' + 9y = 0$

$y = e^{rt}$: $y' = re^{rt}$, $y'' = r^2 e^{rt}$

$y'' + 9y' + 9y = r^2 e^{rt} + 9re^{rt} + 9e^{rt} = (r^2 + 9r + 9)e^{rt} = 0$

since $e^{rt} > 0$ for all t the only option is

$$r^2 + 9r + 9 = 0.$$

By the quadratic formula:

$$r = \tfrac{1}{2}\left(+9 \pm \sqrt{9^2 - 4\cdot 9}\right) = \tfrac{1}{2}\left(9 \pm \sqrt{45}\right)$$
$$= \tfrac{1}{2}\left(9 \pm 3\sqrt{5}\right)$$

so $y_1 = e^{\frac{9+3\sqrt{5}}{2}t}$ and $y_2 = e^{\frac{9-3\sqrt{5}}{2}t}$.

The general solution is $\boxed{y = c_1 e^{\frac{9+3\sqrt{5}}{2}t} + c_2 e^{\frac{9-3\sqrt{5}}{2}t}.}$

#17. If a DE has general solution $y = c_1 e^{2t} + c_2 e^{-3t}$

we know it's characteristic equation must be $(r-2)(r+3) = 0$

Expanding: $r^2 + r - 6 = 0$ so the DE could be $\boxed{y'' + y' - 6y = 0.}$

#24. $y'' + (3-\alpha)y' - 2(\alpha - 1)y = 0$

$r^2 + (3-\alpha)r - 2(\alpha-1) = 0 \Rightarrow r = \tfrac{1}{2}\left(-(3-\alpha) \pm \sqrt{(3-\alpha)^2 + 8(\alpha-1)}\right)$

$$= \tfrac{1}{2}\left(\alpha - 3 \pm \sqrt{9 - 6\alpha + \alpha^2 + 8\alpha - 8}\right)$$
$$= \tfrac{1}{2}\left(\alpha - 3 \pm \sqrt{1 + 2\alpha + \alpha^2}\right)$$
$$= \tfrac{1}{2}\left(\alpha - 3 \pm \sqrt{(1+\alpha)^2}\right)$$
$$= \tfrac{1}{2}\left(\alpha - 3 \pm (1+\alpha)\right)$$
$$= \tfrac{1}{2}(2\alpha - 2),\ \tfrac{1}{2}(-3-1) = \alpha - 1,\ -2.$$

To have all solutions approach zero, both roots must be negative:

$\alpha - 1 < 0 \Leftrightarrow \underline{\alpha < 1}$ $(-2 < 0$ for all $\alpha)$.

Even if $\alpha > 1$ (so one root is positive and one is negative) the solutions that do not include the solution with positive root do not become unbounded.

$\therefore$ For no value of α are <u>all</u> solutions unbounded.