

§2-2 Homework Solution

$$\#7. \frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$$

This is a separable DE: $\int (y + e^y) dy = \int (x - e^{-x}) dx$

$$\frac{1}{2}y^2 + e^y = \frac{1}{2}x^2 + e^{-x} + C.$$

$$\#10 a. y' = \frac{1-2x}{y}, y(1) = -2.$$

This is a separable DE: $\int y dy = \int (1-2x) dx$

$$\frac{1}{2}y^2 = x - x^2 + C$$

Apply the IC: $\frac{1}{2}(-2)^2 = 1 - (1)^2 + C$

$$2 = C$$

so $\frac{1}{2}y^2 = x - x^2 + 2.$

$$y^2 = 2(2 + x - x^2)$$

$$y = -\sqrt{2(2 + x - x^2)} = -\sqrt{2} \sqrt{2-x} \sqrt{1+x}$$

c. To determine the interval of existence, the solution formula tells us that the solution exists so long as $2 + x - x^2 > 0$. Factoring, we see that $(2-x)(1+x) \geq 0$ is needed; that is $-1 \leq x \leq 2$.