MATH 520 (Section 001)
Prof. Meade / Meade

University of South Carolina
Spring 2013

Exam 3
April 17, 2012

Name: ____________________
SS # (last 4 digits): __________

Instructions:

1. There are a total of 5 problems on 2 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) <u>and</u> you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	20	
2	20	
3	16	
4	20	
5	24	
Total	100	

Good Luck!

1. (20 points) Consider the vectors $\mathbf{x}^{(1)}(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$.

   (a) [4 points] Compute the Wronskian of $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$.

   (b) [4 points] In what intervals are $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ linearly independent?

   (c) [4 points] What conclusion can be drawn about the coefficients in the linear system of homogeneous differential equations, $\mathbf{x}' = P(t)\mathbf{x}$, satisfied by $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$?

   (d) [8 points] Find this system of equations and verify the conclusions of part (c).

2. (20 points) Find the solution of the initial value problem $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$.

3. (16 points) Consider $\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}$. Find the general solution for this system.

   Note: The only eigenvalue of A is $\lambda = -3$, with corresponding eigenvector $\boldsymbol{\xi} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

4. (20 points) Consider the nonhomogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$.

   Find the general solution.

   Note: For $\begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$, $\lambda_1 = -3$ with $\boldsymbol{\xi}^{(1)} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and $\lambda_2 = 2$ with $\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

5. (24 points) Each of the following systems, $\mathbf{x}' = A\mathbf{x}$, has an equilibrium solution at the origin. Use the given eigenvalues and eigenvectors to determine the type of this equilibrium solution (including its stability) and to draw representative solutions in the phase plane. (Be sure to clearly label the question you are answering.)

   (a) $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$, $\lambda_1 = 1$, $\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\lambda_2 = -1$, $\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

   (b) $A = \begin{pmatrix} \frac{-7}{3} & \frac{2}{3} \\ \frac{-2}{3} & \frac{-2}{3} \end{pmatrix}$, $\lambda_1 = -1$, $\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\lambda_2 = -2$, $\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (c) $A = \begin{pmatrix} \frac{7}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix}$, $\lambda_1 = 1$, $\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\lambda_2 = 2$, $\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 2 & -2 \\ 4 & -2 \end{pmatrix}$, $\lambda_{1,2} = \pm 2i$, $\boldsymbol{\xi}^{(1),(2)} = \begin{pmatrix} 1 \pm i \\ 2 \end{pmatrix}$

   (e) $A = \begin{pmatrix} 2 & -2 \\ \frac{1}{2} & 0 \end{pmatrix}$, $\lambda_{1,2} = 1$, $\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (f) $A = \begin{pmatrix} \frac{4}{3}\sqrt{6} & \frac{-5}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{6} & \frac{2}{3}\sqrt{6} \end{pmatrix}$, $\lambda_{1,2} = \sqrt{6} \pm i$, $\boldsymbol{\xi}^{(1),(2)} = \begin{pmatrix} 1 \pm \frac{\sqrt{6}}{2} i \\ 1 \end{pmatrix}$